

Solving Systems of Equations by Elimination

Step 1: Write one equation under the other – aligning like variables & constants

Step 2: Multiply every term in one or both equations so that adding the 2 equations eliminates one of the variables.

Step 3: Solve for the remaining variable.

Step 4: Substitute that value into either equation to solve for the other variable.

Ex #1:

$$\begin{cases} 3x + 2y = 4 \\ 4x - 2y = -18 \end{cases}$$

$$7x = -14$$

$$x = -2$$

$$\begin{aligned} 3(-2) + 2y &= 4 \\ -6 + 2y &= 4 \end{aligned}$$

$$2y = 10$$

$$y = 5$$

Solution:  
(-2, 5)

Ex #2:

$$\begin{cases} 2x + y = 7 \\ 3x - y = -12 \end{cases}$$

$$5x = -5$$

$$x = -1$$

step 4

$$2x + y = 7$$

$$2(-1) + y = 7$$

$$-2 + y = 7$$

$$y = 9$$

Solution:  
(-1, 9)

Ex #3:

$$\begin{cases} -2x + y = 5 \\ 10x - 2y = -16 \end{cases} \Rightarrow \begin{aligned} -4x + 2y &= 10 \\ +10x - 2y &= -16 \end{aligned}$$

$$6x = -6$$

$$x = -1$$

step 4:

$$-2x + y = 5$$

$$-2(-1) + y = 5$$

$$2 + y = 5$$

$$y = 3$$

Solution:  
(-1, 3)

Ex #4:

$$\begin{cases} 8x + y = 27 \\ -3x + 4y = 3 \end{cases} \Rightarrow \begin{aligned} -32x - 4y &= -108 \\ + -3x + 4y &= 3 \end{aligned}$$

$$-35x = -105$$

$$x = 3$$

step 4:

$$8x + y = 27$$

$$8(3) + y = 27$$

$$24 + y = 27$$

$$y = 3$$

Solution:  
(3, 3)

Ex #5:

$$\begin{cases} 3x + 5y = -16 \\ 3y + 2x = -9 \end{cases} \Rightarrow \begin{cases} 3x + 5y = -16 \\ -3(2x + 3y = -9) \end{cases} \Rightarrow \begin{array}{r} 6x + 10y = -32 \\ + \quad -6x - 9y = 27 \\ \hline \end{array}$$

$$y = -5$$

$$\begin{aligned} 3x + 5y &= -16 \\ 3x + 5(-5) &= -16 \\ 3x - 25 &= -16 \\ 3x - 25 + 25 &= -16 + 25 \end{aligned}$$

$$\begin{aligned} 3x &= 9 \\ \frac{3x}{3} &= \frac{9}{3} \\ x &= 3 \end{aligned}$$

Solution:  
(3, -5)

Special Systems:

$$\begin{cases} y - 3x = 4 \\ 2y - 6x = 8 \end{cases} \Rightarrow \begin{array}{r} -2y + 6x = -8 \\ + \quad 2y - 6x = 8 \\ \hline \end{array}$$

$$\begin{cases} 2x - y = 3 \\ 4x - 2y = -12 \end{cases} \Rightarrow \begin{array}{r} -4x + 2y = -6 \\ + \quad 4x - 2y = -12 \\ \hline \end{array}$$

0 = 0 true statement!  
Same line!  
Infinite solutions

0 = -18 false!  
no solution  
means lines parallel w/ d.f. y intercepts

Word problems!

Ex: Marcus is thinking of two numbers whose sum is 89. Their difference is 11. What are the two numbers?

$$\begin{array}{r} x + y = 89 \\ + \quad x - y = 11 \\ \hline 2x = 100 \\ x = 50 \end{array} \quad \begin{array}{r} x + y = 89 \\ 50 + y = 89 \\ -50 \quad -50 \\ \hline y = 39 \end{array}$$

Ex: To raise money for new uniforms, Hy-Line is selling t-shirts and hats. They spent \$2000 purchasing the merchandise and brought in \$3375 in total sales. How many t-shirts did they sell?

Item	Cost	Sale price
T-shirt	\$6	\$10
hat	\$4	\$7

$$\begin{cases} 6x + 4y = 2000 \\ 10x + 7y = 3375 \end{cases} \Rightarrow \begin{array}{r} -30x - 20y = -10,000 \\ 3(10x + 7y = 3375) \\ \hline \end{array}$$

y = 125 hats sold  
x = 250 t-shirts sold

$$\begin{aligned} 6x + 4y &= 2000 \\ 6x + 4(125) &= 2000 \\ 6x + 500 &= 2000 \\ 6x &= 1500 \\ x &= \frac{1500}{6} = 250 \end{aligned}$$