Name: Key Period: ______ 2018

Algebra II 2016 Ch. 3.5 – 3.8 Test Review

3.5 Matrix Basics and Addition, Subtraction and Scalar Multiplication – Four Function Calc

1. What are the dimensions of A? What are the dimensions of B?

$$A = \begin{bmatrix} 2 & -1 & 0 & 2 \\ -3 & 6 & 8 & 2 \\ 0 & -3 & -5 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 \\ -1 & 3 \\ 7 & -2 \\ 0 & -4 \\ 1 & -2 \end{bmatrix}$$

3×4 5×2

a_{23} = 8 address of -4 in matrix B = ~~4×~~ b_{42}

2. If $C = \begin{bmatrix} -2 & -3 & 4 \\ 1 & 2 & -1 \\ 0 & -7 & -5 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & -6 & 3 \\ -2 & 3 & 8 \\ 5 & 2 & 0 \end{bmatrix}$, find C – D.

$$\begin{bmatrix} -2 & -3 & 4 \\ 1 & 2 & -1 \\ 0 & -7 & -5 \end{bmatrix} + \begin{bmatrix} -1 & 6 & -3 \\ 2 & -3 & -8 \\ -5 & -2 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 3 & 1 \\ 3 & -1 & -9 \\ -5 & -9 & -5 \end{bmatrix}$$

3. If $K = \begin{bmatrix} -2 & 0 & 7 \\ 10 & 4 & -1 \end{bmatrix}$ and $S = \begin{bmatrix} 5 & 6 \\ 16 & -2 \\ 1 & 4 \end{bmatrix}$, find K + S

2×3 3×2

Not possible

4. If $M = \begin{bmatrix} 3 & 2 \\ -6 & 4 \\ -1 & -5 \end{bmatrix}$ Evaluate the expression 3M

$$3M = \begin{bmatrix} 9 & 6 \\ -18 & 12 \\ -3 & -15 \end{bmatrix}$$

5. If $C = \begin{bmatrix} -1 & 8 \\ 2 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} -3 & -6 \\ 8 & 1 \end{bmatrix}$, find 3D – 2C.

$$\begin{bmatrix} -9 & -18 \\ 24 & 3 \end{bmatrix} + \begin{bmatrix} +2 & -16 \\ -4 & -6 \end{bmatrix} = \begin{bmatrix} -7 & -34 \\ 20 & -3 \end{bmatrix}$$

3.6 Matrix Multiplication – <u>Four Function Calculator</u>

1. Given $A = \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 0 \end{bmatrix}$, evaluate AB and BA.

3 × 1 1 × 3

AB
3 × 1 1 × 3

$$\begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -3 & 0 \\ 8 & -4 & 0 \\ -4 & 2 & 0 \end{bmatrix}$$

3 × 3

BA
1 × 3 3 × 1

$$\begin{bmatrix} 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 0 \end{bmatrix}$$

1 × 1

2. Evaluate the matrix expressions CD and DC.

$C = \begin{bmatrix} 6 \\ -2 \\ -5 \end{bmatrix}$ $D = \begin{bmatrix} 3 & -2 & 1 \\ -6 & 2 & 0 \end{bmatrix}$

3 × 1 2 × 3

CD – Not possible

DC
2 × 3 3 × 1

$$\begin{bmatrix} 3 & -2 & 1 \\ -6 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ -2 \\ -5 \end{bmatrix} = \begin{bmatrix} 17 \\ -40 \end{bmatrix}$$

3. A car lot has four sales associates. At the end of the year, each sales associate gets a bonus of $1000 for every new car they have sold and $500 for every used car they have sold.

a. Write a matrix for the number of each type of car each sales associate sold this year.

Cars Sold by Each Associate		
Sales Associate	New Cars	Used Cars
Mason	27	49
Westin	35	36
Gallagher	9	56
Stadler	15	62

New Used
M [27 49]
W [35 36]
G [9 56]
S [15 62]

b. Then write a bonus amount matrix.

Bonus
New [1000]
Used [500]

c. Use matrix multiplication to find the total bonus each sales associate earned. Label the rows and columns of the new matrix.

$$\begin{bmatrix} 27 & 49 \\ 35 & 36 \\ 9 & 56 \\ 15 & 62 \end{bmatrix} \cdot \begin{bmatrix} 1000 \\ 500 \end{bmatrix} = \begin{bmatrix} \$51{,}500 \\ 53{,}000 \\ 37{,}000 \\ 46{,}000 \end{bmatrix}$$

Rows: Mason, Westin, Gallagher, Stadler; columns: New, Used; Bonus

4 x 2 · 2 x 1 = 4 x 1

d. Which associate earned the most money? Which associate earned the least?

Mason Gallagher

e. What was the total amount of money the car lot spend on bonuses for all the sales associates?

51,500 + 53000, 37000 + 46000 = $187,500

3.8 Part 1 Matrix Inverses – Four Function Calculator

Multiple. If answer matrix is $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ then yes!

Determine whether the matrices in each pair are inverses of each other.

1. $\begin{bmatrix}7 & 2\\17 & 5\end{bmatrix}$ and $\begin{bmatrix}5 & -2\\-17 & 7\end{bmatrix}$

$\begin{bmatrix}7 & 2\\17 & 5\end{bmatrix} \cdot \begin{bmatrix}5 & -2\\-17 & 7\end{bmatrix}$

$\begin{bmatrix}35-34 & -14+14\\85-85 & -34+35\end{bmatrix}$

Yes inverses $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

2. $\begin{bmatrix}3 & 2\\4 & -6\end{bmatrix}$ and $\begin{bmatrix}3 & 2\\-4 & -3\end{bmatrix}$

$\begin{bmatrix}9+-8 & 6-6\\12+24 & 8+18\end{bmatrix}$

$\begin{bmatrix}1 & 0\\36 & 26\end{bmatrix}$

No

3. $\begin{bmatrix}5 & 2\\11 & 4\end{bmatrix}$ and $\begin{bmatrix}-2 & 1\\\frac{11}{2} & -\frac{5}{2}\end{bmatrix}$

$\begin{bmatrix}-10+11 & 5-5\\-22+22 & 11-10\end{bmatrix}$

$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

yes!

Find the inverse of the given matrix.

$A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$	det A = $ad - bc$	Inverse of A: $A^{-1} = \frac{1}{\det A}\begin{bmatrix}d & -b\\-c & a\end{bmatrix}$

4. $\begin{bmatrix}5 & 8\\4 & 6\end{bmatrix}$

$\det A = 5\cdot 6 - 4\cdot 8$
$= 30 - 32$
$= -2$

$A^{-1} = \frac{1}{-2}\begin{bmatrix}6 & -8\\-4 & 5\end{bmatrix}$

$= \begin{bmatrix}-3 & 4\\2 & -\frac{5}{2}\end{bmatrix}$

5. $\begin{bmatrix}4 & 3\\7 & 5\end{bmatrix}$

$\det A =$
$4\cdot 5 - 7\cdot 3$
$20 - 21$
1

$A^{-1} = 1 \cdot \begin{bmatrix}5 & -3\\-7 & 4\end{bmatrix}$

$= \begin{bmatrix}5 & -3\\-7 & 4\end{bmatrix}$

6. $\begin{bmatrix}5 & -10\\3 & -6\end{bmatrix}$

$\det A =$
$5(-6) - 3(-10)$
$-30 + 30$
0

$\frac{1}{0}\begin{bmatrix}-6 & +10\\-3 & 5\end{bmatrix}$

Not possible

Inverse doesn't exist

Singular matrix

7. If A = $\begin{bmatrix} 2 & -3 \\ -1 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} x \\ y \end{bmatrix}$ find AB

$$\begin{bmatrix} 2 & -3 \\ -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x-3y \\ -x+5y \end{bmatrix}$$

2×2 $\quad 2 \times 1$ $\quad 2 \times 1$

8. If A = $\begin{bmatrix} 5 & -8 \end{bmatrix}$ and B = $\begin{bmatrix} x \\ y \end{bmatrix}$ find AB $\quad \begin{bmatrix} 5x-8y \end{bmatrix}$

1×2 $\quad 2 \times 1$ $\quad 1 \times 1$

3.8 Part 2 – Systems with matrices – Graphing Calculators Allowed

For each system, write out the matrix equation and then solve. Don't forget to put each equation in standard form first!

1. $\begin{cases} 5x = 8 + 3y \\ 6x - 4 = 5y \end{cases}$ $\quad$ $5x - 3y = 8$, $\quad 6x - 5y = 4$

Two lines
They intersect at

Matrix Equation: $\begin{bmatrix} 5 & -3 \\ 6 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

$A \cdot X = B$

$X = A^{-1} \cdot B = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

Solution: (4, 4)

a point! 1 soln ☺

2. $\begin{cases} 5y = -4x \\ 5x + 3y = 13 \end{cases}$ $\quad$ $4x + 5y = 0$, $\quad 5x + 3y = 13$

Two lines.
Intersect at

Matrix Equation: $\begin{bmatrix} 4 & 5 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 13 \end{bmatrix}$

$A \cdot X = B$

$X = A^{-1}B = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

Solution: (5, -4)

one soln!

Standard form 1st!

3. $\begin{cases} 3x = 5 - 4z \\ x + y + z = 5 \\ y = 2z \end{cases}$

$3x + 0y + 4z = 5$
$x + y + z = 5$
$0x + y - 2z = 0$

Matrix Equation: $\begin{bmatrix} 3 & 0 & 4 \\ 1 & 1 & 1 \\ 0 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix}$

$A \cdot X = B$

$X = A^{-1} \cdot B = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$

Solution: $(-1, 4, 2)$ ← an ordered triple, one pt soln is 3-space!

4. $\begin{cases} x + y = z \\ 5y - 2z = 4 \\ 5y - 2x = 8 \end{cases}$

$x + y - z = 0$
$0x + 5y - 2z = 4$
$-2x + 5y + 0z = 8$

Matrix Equation: $\begin{bmatrix} 1 & 1 & -1 \\ 0 & 5 & -2 \\ -2 & 5 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 8 \end{bmatrix}$

$A \cdot X = B$

$X = A^{-1} B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Solution: $(1, 2, 3)$

on soln, an ordered triple in 3 space.

5. Keith paid $39 for 3 pounds of pistachios and 2 pounds of cashews. Tracey paid $23 for 2 pounds of pistachios and 1 pound of cashews. How much does a pound of each type of nut cost?

a. Write a system of equations.
Let x = the cost of a pound of pistachios,
and y = the cost of a pound of cashews.

$3x + 2y = 39$
$2x + y = 23$

b. Write the matrix equation & solve for the cost of each type of nut.
Remember to write your solution in word.

$A \cdot X = B$

Matrix Equation: $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 39 \\ 23 \end{bmatrix}$

$X = A^{-1} \cdot B = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$

Solution: $(7, 9)$

Interpretation: pistachios cost $7/lb, cashews cost $9/lb

6. Frank and Juanita sold tickets for the charity fund-raiser. They sold both single tickets and 5-ticket books. Write the appropriate matrix equation and find the price of a single ticket and a book of tickets.

X = cost of single ticket

y = cost of 5-ticket books

Fund-Raiser Tickets Sold			
	Single	Book	Total Sales
Frank	12	4	70
Juanita	8	3	50

$12x + 4y = 70$

$8x + 3y = 5$

$$\begin{bmatrix} 12 & 4 \\ 8 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 70 \\ 50 \end{bmatrix}$$

$A \cdot X = B$

$X = A^{-1}B$

$$\begin{bmatrix} 2.5 \\ 10 \end{bmatrix}$$

$2.50 for a single ticket

$10 for 5-ticket book

7. Theo paid $89 for 3 T-shirts, a sweatshirt, and a jacket. Andre paid $49 for 2 T-Shirts and a jacket. Jordan paid $68 for 1 T-shirt and 2 sweatshirts. What is the price of a single sweatshirt?

X = cost of T-shirt

Y = " " sweatshirt

Z = " " jacket

$3x + y + z = 89$

$2x + 0y + z = 49$

$x + 2y + 0z = 68$

$$\begin{bmatrix} 3 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 89 \\ 49 \\ 68 \end{bmatrix}$$

$A \cdot X = B$

$X = A^{-1} \cdot B$

$$= \begin{bmatrix} 12 \\ 28 \\ 25 \end{bmatrix}$$

Single Sweatshirt price: $28