

## 3.6 Multiplying Matrices

A MATRIX IS A RECTANGULAR ARRAY OF NUMBERS ENCLOSED IN BRACKETS. ITS DIMENSIONS ARE DESCRIBED AS ROW x COLUMN. EACH VALUE IN A MATRIX IS CALLED AN ENTRY.

RULES FOR MULTIPLYING MATRICES:

1. The number of columns in the first matrix must be the same as the number of rows in the second matrix.
2. The product of  $m \times n$  and  $n \times p$  is a matrix with dimensions  $m \times p$ .

$A_{2 \times 3} = \begin{bmatrix} 3 & 5 & 7 \\ 4 & 1 & 2 \end{bmatrix}$ 
 $B_{3 \times 4} = \begin{bmatrix} 2 & 3 & 3 & 8 \\ 9 & 5 & 2 & 0 \\ 0 & 1 & 6 & 7 \end{bmatrix}$ 
 Dimensions of A and B:  $2 \times 3$  and  $3 \times 4$ . Answer should be \_\_\_\_\_.

matrix notation

Determine if it is possible to multiply the following matrices. If so, determine the dimensions of the resulting matrix.

$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ 
 $B = \begin{bmatrix} 4 & 6 \\ 3 & 15 \end{bmatrix}$ 
 $C = \begin{bmatrix} 2 & 4 & 3 & 8 \\ 9 & 5 & -2 & 0 \\ 0 & 1 & -6 & 7 \end{bmatrix}$ 
 $D = \begin{bmatrix} 4 & 6 & 5 \\ 3 & 15 & 8 \end{bmatrix}$ 
 $E = \begin{bmatrix} 2 & 14 \\ 7 & -3 \\ 0 & 1 \end{bmatrix}$

1) AD  
 $2 \times 2$   $2 \times 3$   
 yes result =  $2 \times 3$  matrix

2) DA  
 $2 \times 3$   $2 \times 2$   
 no!

3) CB  
 $3 \times 4$   $2 \times 2$   
 no!

4) DC  
 $2 \times 3$   $3 \times 4$  yes!  
 =  $2 \times 4$  matrix

5) ED  
 $3 \times 2$   $2 \times 3$  yes!  
 result =  $3 \times 3$  matrix

6) DE  
 $2 \times 3$   $3 \times 2$   
 yes! result =  $2 \times 2$  matrix

To multiply matrices you multiply the 1<sup>st</sup> row in A by the 1<sup>st</sup> column in B! Add the sums of the products!

GIVEN:

$A = \begin{bmatrix} 0 & 4 & 9 \\ -3 & 3 & 2 \end{bmatrix}$ 
 $B = \begin{bmatrix} 5 & 11 \\ -2 & 7 \\ -6 & 0 \end{bmatrix}$ 
 $C = \begin{bmatrix} 11 & -1 \\ 12 & -10 \end{bmatrix}$

Find the following products, if possible.

1. AB  
 $2 \times 3$   $3 \times 2$  result =  $2 \times 2$

$$\begin{bmatrix} 0 & -4+54 & 0+28+0 \\ -3+6+12 & -3+21+0 \end{bmatrix} = \begin{bmatrix} 46 & 28 \\ -9 & 18 \end{bmatrix}$$

2. BC

$$\begin{array}{c}
 \text{B} \quad \text{C} \\
 \left[ \begin{array}{cc} 5 & 1 \\ -2 & 7 \\ 6 & 0 \end{array} \right] \cdot \left[ \begin{array}{cc} 11 & -1 \\ 12 & 10 \end{array} \right] = \left[ \begin{array}{cc} \underline{55+12} & \underline{-5+10} \\ \underline{-22+84} & \underline{2+70} \\ \underline{66+0} & \underline{-6+0} \end{array} \right] \\
 = \left[ \begin{array}{cc} 67 & 5 \\ 62 & 72 \\ 66 & -6 \end{array} \right]
 \end{array}$$

3. CB

$$\left[ \begin{array}{cc} 11 & -1 \\ 12 & 10 \end{array} \right]_{2 \times 2} \cdot \left[ \begin{array}{cc} 5 & 1 \\ -2 & 7 \\ 6 & 0 \end{array} \right]_{3 \times 2} \quad \text{not possible}$$

NEED TO KNOW:

A square matrix has the same number of rows and columns. The main diagonal is the diagonal from the upper left corner to the lower right corner.

$$\begin{bmatrix} 2 & 3 \\ 9 & 12 \end{bmatrix}$$

Square matrices are the only matrices that you can raise to a power!!

$$A = \begin{bmatrix} 7 & 1 \\ 2 & 0 \end{bmatrix} \quad \text{Find } A^2$$

$$\begin{array}{c}
 \left[ \begin{array}{cc} 7 & 1 \\ 2 & 0 \end{array} \right] \cdot \left[ \begin{array}{cc} 7 & 1 \\ 2 & 0 \end{array} \right] = \left[ \begin{array}{cc} \underline{49+2} & \underline{7+0} \\ \underline{14+0} & \underline{2+0} \end{array} \right] \\
 = \left[ \begin{array}{cc} 51 & 7 \\ 14 & 2 \end{array} \right]
 \end{array}$$

$$C = \begin{bmatrix} 1 & 0 & 2 \\ 4 & 9 & 5 \end{bmatrix} \quad \text{Can you find } C^2? \text{ Why or why not?}$$

not possible

$$\begin{array}{c}
 C \cdot C \\
 2 \times 3 \quad 2 \times 3 \\
 \underbrace{\hspace{2cm}} \\
 2
 \end{array}$$

inner dimensions  
don't match