Day 1

3.8 MATRIX INVERSES AND MATRIX EQUATIONS

Warm-up:

$M = [2 \quad 4 \quad 6]$ $\qquad N = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$

Multiply the following if possible:

MN

$\underbrace{1\times 3 \; 3\times 1}$ $= 1\times 1$ matrix

$$[2 \; 4 \; 6]\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 2(1)+4(3)+6(5) \end{bmatrix} = [44]$$

NM

$$\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}[2 \; 4 \; 6] = \begin{bmatrix} 2 & 4 & 6 \\ 6 & 12 & 18 \\ 10 & 20 & 30 \end{bmatrix}$$

$3\times 1 \;\; 1\times 3$

$= 3\times 3$ matrix

WHAT IS AN INVERSE? matrices that when multiplied together result in the identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

SO WHEN DOES AN INVERSE MATRIX EXIST?

Only $\boxed{\text{square}}$ matrices can have inverses, but not all do. If the product of a square matrix and its inverse result in the identity matrix then the inverse exists.

$$IF\ AA^{-1} = A^{-1}A = I$$
$$\text{THEN } A^{-1} \text{ IS THE INVERSE OF A}$$

SO WHAT IS AN IDENTITY MATRIX?

Regular multiplication has an identity value: 1. When you multiply by 1 the value does not change. We need a corresponding matrix that has the same property.

The identity matrix is different for each size of square matrix:

$$I_{2x2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_{3x3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

main diagonal top left to bottom right

An identity matrix has 1's along the main diagonal and 0's everywhere else. If two matrices are inverses of one another, their product will produce the identity matrix that equals one.

EX: Use multiplication to determine whether the two given matrices are inverses of one another.

1. $A = \begin{bmatrix} 2 & 3 \\ 7 & 10 \end{bmatrix}$ *and* $B = \begin{bmatrix} -10 & 6 \\ 7 & -4 \end{bmatrix}$ $\begin{bmatrix} 2(-10)+3(7) & 2(6)+3(-4) \\ 7(-10)+10(7) & 7(6)+10(-4) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ no!

2. $F = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ *and* $G = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$ $\begin{bmatrix} -1(-1)+1(0) & -1(-1)+1(-1) \\ 0(-1)+-1(0) & 0(-1)+-1(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ yes!

HOW DO WE FIND AN INVERSE MATRIX?

Inverse matrices use the determinant to generate the inverse matrix. The determinant of a 2 x 2 matrix is the difference in the products of the main diagonal (top left to bottom right) and the secondary diagonal.

INVERSE OF A 2 x 2 MATRIX:

Inverse = $\frac{1}{\det}$ • modified matrix - switch main diagonal & change signs on secondary.

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ **Inverse of A:** $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

If the determinant = 0 then the matrix has no inverse and it is called a singular matrix.

EX: Find the inverses for each matrix below.

1. $A = \begin{bmatrix} -2 & 2 \\ 3 & -4 \end{bmatrix}$

Step 1: det: $-2(-4) - 3(2)$
$= 8 - 6 = 2$

Step 2: $\frac{1}{2}\begin{bmatrix} -4 & -2 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -3/2 & -1 \end{bmatrix}$ ← Inverse

Check: $\begin{bmatrix} -2 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ -3/2 & -1 \end{bmatrix} = \begin{bmatrix} 4-3 & 2-2 \\ -6+6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ yes!

2. $D = \begin{bmatrix} 2 & -3 \\ 6 & -9 \end{bmatrix}$

determinant: $(2)(-9) - (-3)(6)$
$-18 - (-18)$
$= -18 + 18 = 0$
Singular matrix b/c d = 0

3. $C = \begin{bmatrix} 6 & -3 \\ -1 & 0 \end{bmatrix}$

Step 1 det: $0 - 3$
$= -3$

Step 2: $-\frac{1}{3}\begin{bmatrix} 0 & 3 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1/3 & -2 \end{bmatrix}$
inverse

Step 3: Check $\begin{bmatrix} 0 & -1 \\ -1/3 & -2 \end{bmatrix}\begin{bmatrix} 6 & -3 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+1 & 0+0 \\ -2+2 & 1+0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ yes!