3.8 Day 2

SOLVING SYTEMS USING MATRIX EQUATIONS

Warm-up:

If $A = \begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix}$ *and* $B = \begin{bmatrix} x \\ y \end{bmatrix}$ find AB.

$2 \times 2 \quad 2 \times 1$

$= \begin{bmatrix} 4x+7y \\ 3x+5y \end{bmatrix}$

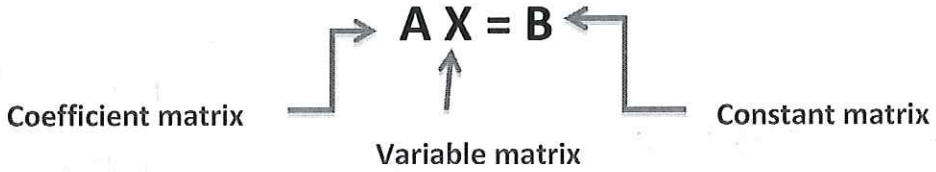
$\begin{bmatrix} 4x+7y \\ 3x+5y \end{bmatrix}$

An inverse matrix can be used to solve a system of equations, very similar to how you use an inverse in an algebraic equation.

Think of the equation $3x = 9$. To solve you multiply both sides by the inverse of 3!

Now consider the system $\begin{cases} 4x + 7y = 10 \\ 3x + 5y = 9 \end{cases}$

To solve using matrices, first we set up a Matrix Equation:



Matrix Equation for the system above:

$$\begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 9 \end{bmatrix}$$

The solution is found by multiplying the inverse of the coefficient matrix by the constant matrix! Since multiplication of matrices is NOT commutative you must multiply by A^{-1} FIRST on both sides.

Formula: $A^{-1}B = X$

Let's solve this system: (This is how you would show your work.)

$$\begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 9 \end{bmatrix}$$

A B X

$A^{-1}B = \begin{bmatrix} 13 \\ -6 \end{bmatrix}$

$= (13, -6)$

$x \quad y$

For each system, write the matrix equation and then solve.

Ex #1:

$\begin{cases} 5x + 2y = 20 \\ 3y + 6x = 9 \end{cases}$ → Rewrite! $5x+2y=20$, $6x+3y=9$

Matrix Equation: $\underset{A}{\begin{bmatrix} 5 & 2 \\ 6 & 3 \end{bmatrix}} \cdot \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 20 \\ 9 \end{bmatrix}}$ $\quad A^{-1}B = \begin{bmatrix} x \\ y \end{bmatrix}$

Solution: $(14, -25)$

Ex #2:

$\begin{cases} 2x + 6y = 5 \\ -x + 2z = 4y + 2 \\ 3z = 5y - 7x \end{cases}$ → $2x+6y+0z=5$, $-x-4y+2z=2$, $7x-5y+3z=0$ $\quad A^{-1}B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Matrix Equation: $\underset{A}{\begin{bmatrix} 2 & 6 & 0 \\ -1 & -4 & 2 \\ 7 & -5 & 3 \end{bmatrix}} \cdot \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix}}$

Solution: $\begin{bmatrix} -\frac{23}{49} & \frac{97}{98} & \frac{269}{98} \end{bmatrix}$

Ex #3:

A game show host says that he has $5000 in $50 bills and $100 bills and he will give you the $5000 if you can tell him how many of each type of bill he has. He gives you a hint that he has 73 bills. Find out how many of each he has.

let x = # \$50 bills
let y = # \$100 bills
73 total

$x + y = 73$
$50x + 100y = \$5000$

System of Equations: _____

Matrix Equation: $\underset{A}{\begin{bmatrix} 1 & 1 \\ 50 & 100 \end{bmatrix}} \cdot \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 73 \\ 5000 \end{bmatrix}}$ $\quad A^{-1} \cdot B = X$

Solution: $(46, 27)$

answer: There are 46 fifty dollar bills
There are 27 one hundred dollar bills