

## SOLVING SYTEMS USING MATRIX EQUATIONS

Warm-up:

If  $A = \begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} x \\ y \end{bmatrix}$  find  $AB$ .

An inverse matrix can be used to solve a system of equations, very similar to how you use an inverse in an algebraic equation.

Think of the equation  $3x = 9$ . To solve you multiply both sides by the inverse of 3!

Now consider the system  $\begin{cases} 4x + 7y = 10 \\ 3x + 5y = 9 \end{cases}$

To solve using matrices, first we set up a Matrix Equation:

$$\begin{array}{ccc} & \begin{array}{c} \text{Coefficient matrix} \end{array} \rightarrow & \mathbf{A X = B} & \leftarrow \begin{array}{c} \text{Constant matrix} \end{array} \\ & \begin{array}{c} \uparrow \\ \text{Variable matrix} \end{array} & & \end{array}$$

**Matrix Equation for the system above:**

$$\begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 9 \end{bmatrix}$$

The solution is found by multiplying the inverse of the coefficient matrix by the constant matrix! Since multiplication of matrices is NOT commutative you must multiply by  $A^{-1}$  FIRST on both sides.

Formula:  $A^{-1}B = X$

Let's solve this system: (This is how you would show your work.)

$$\begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 9 \end{bmatrix}$$

For each system, write the matrix equation and then solve.

Ex #1:

$$\begin{cases} 5x + 2y = 20 \\ 3y + 6x = 9 \end{cases}$$

Matrix Equation: \_\_\_\_\_

Solution: \_\_\_\_\_

Ex #2:

$$\begin{cases} 2x + 6y = 5 \\ -x + 2z = 4y + 2 \\ 3z = 5y - 7x \end{cases}$$

Matrix Equation: \_\_\_\_\_

Solution: \_\_\_\_\_

Ex #3:

A game show host says that he has \$5000 in \$50 bills and \$100 bills and he will give you the \$5000 if you can tell him how many of each type of bill he has. He gives you a hint that he has 73 bills. Find out how many of each he has.

System of Equations: \_\_\_\_\_

Matrix Equation: \_\_\_\_\_

Solution: \_\_\_\_\_