4.3 Notes Day 2

Objectives: Solve Quadratic Equations by Factoring
(Perfect Square, Difference of Squares and Trinomials with a >1)

Trinomials that are perfect squares have special factoring rules. In order to use these rules, the first and last terms need to be perfect squares and the middle term needs to be twice the product of the square roots of the first and last terms.

Ex. $x^2 + 16x + 64 = 0$

$\sqrt{1} = 1$
$\sqrt{64} = 8$
$2(1 \cdot 8)$
$= 2(8) = 16$ ✓

$(x+8)(x+8) = 0$
or $(x+8)^2 = 0$
$x = -8$

Ex. $4x^2 - 12x + 9 = 0$ $= (2x-3)(2x-3) = 0$

$\sqrt{4} = 2$
$\sqrt{9} = 3$
$2(2 \cdot 3) = 2 \cdot 6 = 12$ ✓

or $(2x-3)^2 = 0$
$x = 3/2$

1. $x^2 - 12x + 36 = 0$

$\sqrt{1} = 1$
$\sqrt{36} = 6$
$2(1 \cdot 6)$
$= 12$ ✓

$(x-6)(x-6) = 0$
$(x-6)^2 = 0$
$x = 6$

2. $x^2 + 14x + 49 = 0$

$\sqrt{1} = 1$
$\sqrt{49} = 7$
$2(1 \cdot 7)$
$= 14$ ✓

$(x+7)(x+7) = 0$
$x = -7$

3. $9x^2 + 30x + 25 = 0$

$\sqrt{9} = 3$
$\sqrt{25} = 5$
$2(3 \cdot 5)$
$= 2 \cdot 15 = 30$ ✓

$(3x+5)(3x+5) = 0$
$3x + \cancel{5} = 0$
$\cancel{-5} \; -5$
$\frac{\cancel{3}x}{\cancel{3}} = \frac{-5}{3}$
$x = -5/3$

Binomials that are difference of squares have special factoring rules. In order to use these rules, the two terms need to be subtracted and the two terms need to be perfect squares.

Ex. $x^2 - 64 = 0$

$(x+8)(x-8) = 0$
$x+8=0$ $\quad$ $x-8=0$
$x = -8$ $\quad$ $x = 8$

Ex. $x^2 = 36$

$x^2 - 36 = 0$
$(x+6)(x-6) = 0$
$x+6=0$ $\quad$ $x-6=0$
$x = -6$ $\quad$ $x = 6$

Ex. $x^2 + 100 = 0$

Can't factor!
Not difference of squares

4. $x^2 - 121 = 0$

$(x+11)(x-11) = 0$
$x+11=0$ $\quad$ $x-11=0$
$x = -11$ $\quad$ $x = 11$

5. $4x^2 - 25 = 0$

$(2x+5)(2x-5) = 0$
$2x + \cancel{5} = 0$ $\quad$ $2x - 5 = 0$
$\cancel{-5} \; -5$ $\quad$ $2x = 5$
$\frac{\cancel{2}x}{\cancel{2}} = \frac{-5}{2}$
$x = -\frac{5}{2}$ $\quad$ $x = 5/2$

6. $64x^2 - 81 = 0$

$(8x+9)(8x-9) = 0$
$8x+9=0$ $\quad$ $8x-9=0$
$-9 \;\; -9$ $\quad$ $+9 \;\; +9$
$8x = -9$ $\quad$ $8x = 9$
$x = -\frac{9}{8}$ $\quad$ $x = \frac{9}{8}$

A special pattern is used when factoring trinomials of the form $ax^2 + bx + c$. First, multiply the values of a and c. Then, find two values, m and p, such that their product equals ac and their sum equals b.

Consider: $6x^2 + 13x - 5$

$6 \cdot -5 = -30$

-2	15
-3	10
-6	5

$+ = 13$

Now the middle term, 13x, can be written as $-2x + 15x$.

Rewrite the polynomial replacing the middle term with the new expression.

This new polynomial can now be factored by grouping and then solved.

$6x^2 - 2x + 15x - 5$

$2x(3x-1) + 5(3x-1) = (2x+5)(3x-1) = 0$

$2x+5=0$ $\quad$ $3x-1=0$

$x = -5/2$ $\quad$ $x = 1/3$

Solve the following quadratic equations using the above pattern.

7. $2x^2 + 15x + 7 = 0$

$2 \cdot 7 = 14$

1	14
2	7

$2x^2 + 1x + 14x + 7 = 0$

$x(2x+1) + 7(2x+1) = 0$

$(x+7)(2x+1) = 0$

$x+7=0$ $\quad$ $2x+1=0$

$x = -7$ $\quad$ $2x = -1$

$x = -1/2$

8. $3x^2 - 5x - 12 = 0$

$3 \cdot -12 = -36$

2	-18
3	-12
4	-9

$3x^2 - 9x + 4x - 12 = 0$

$3x(x-3) + 4(x-3) = 0$

$(3x+4)(x-3) = 0$

$3x+4=0$ $\quad$ $x-3=0$

$3x = -4$ $\quad$ $x = 3$

$x = -4/3$

9. $9x^2 + 11x + 2 = 0$

$9 \cdot 2 = 18$

2	9

$9x^2 + 9x + 2x + 2 = 0$

$9x(x+1) + 2(x+1) = 0$

$(9x+2)(x+1) = 0$

$9x+2=0$ $\quad$ $x+1=0$

$9x = -2$ $\quad$ $x = -1$

$x = -2/9$

10. $7x^2 - 22x + 3 = 0$

$7 \cdot 3 = 21$

3	7
1	21

$7x^2 - 21x - x + 3 = 0$

$7x(x-3) - 1(x-3) = 0$

$(7x-1)(x-3) = 0$

$7x-1=0$ $\quad$ $x-3=0$

$7x = 1$ $\quad$ $x = 3$

$x = 1/7$