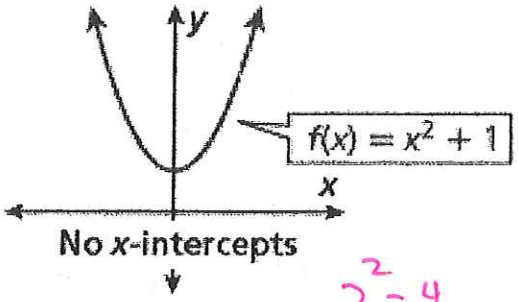
Key

4.4 Notes Day 1

Objectives: What is an imaginary number. How to find square roots of negative numbers. Find products of imaginary numbers. Solve equations involving imaginary numbers.

You can see in the graph of $f(x) = x^2 + 1$ below that f has no real zeros. If you solve the corresponding equation $0 = x^2 + 1$, you find that $x = \pm\sqrt{-1}$, which has no *real* solutions.

However, you can find solutions if you define the square root of negative numbers, which is why *imaginary numbers* were invented. The imaginary unit i is defined as $\sqrt{-1}$. You can use the imaginary unit to write the square root of any negative number.



Imaginary Numbers

WORDS	NUMBERS	ALGEBRA
An imaginary number is the square root of a negative number.	$\sqrt{-1} = i$	If b is a positive real number, then $\sqrt{-b} = i\sqrt{b}$ and $\sqrt{-b^2} = bi$.
Imaginary numbers can be written in the form bi, where b is a real number and i is the imaginary unit.	$\sqrt{-2} = \sqrt{-1}\sqrt{2} = i\sqrt{2}$ $\sqrt{-4} = \sqrt{-1}\sqrt{4} = 2i$	$(\sqrt{-b})^2 = -b$
The square of an imaginary number is the original negative number.	$(\sqrt{-1})^2 = i^2 = -1$	

$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$
$12^2 = 144$
$13^2 = 169$

Express the number in terms of i.

Example A:

$\sqrt{-16}$

$\sqrt{16}\sqrt{-1}$

$= 4i$

$\sqrt{-28}$

$\sqrt{4 \cdot 7}\sqrt{-1}$

$= 2i\sqrt{7}$

$3\sqrt{-24}$

$3\sqrt{4 \cdot 6}\sqrt{-1}$

$3 \cdot 2i\sqrt{6}$

$= 6i\sqrt{6}$

$5\sqrt{-121}$

$5\sqrt{121}\sqrt{-1}$

$5 \cdot 11 \cdot i$

$= 55i$

1) $\sqrt{-27}$

$\sqrt{9 \cdot 3}\sqrt{-1}$

$3i\sqrt{3}$

2) $\sqrt{-216}$

$\sqrt{36 \cdot 6}\sqrt{-1}$

$6i\sqrt{6}$

3) $\sqrt{-18}$

$\sqrt{9 \cdot 2}\sqrt{-1}$

$3i\sqrt{2}$

What does "i" equal? $\sqrt{-1}$ So, what does "i^2" equal? $\sqrt{-1}\cdot\sqrt{-1} = -1$

$i^1 = i$	$i^2 = -1$	$i^3 = i^2 \cdot i$ or $-i$	$i^4 = (i^2)^2$ or 1
$i^5 = i^4 \cdot i$ or i	$i^6 = i^4 \cdot i^2$ or -1	$i^7 = i^4 \cdot i^3$ or $-i$	$i^8 = (i^2)^4$ or 1

Simplify $-6i^{14}$

$-6(i^2)^7$

$= -6(-1)^7$

$= -6(-1) = 6$

Simplify $-5i \cdot 3i$

$-5 \cdot 3 \cdot i \cdot i$

$= -15i^2$

$= -15(-1)$

$= 15$

Simplify $\sqrt{-6} \cdot \sqrt{-15}$

$\sqrt{6}\sqrt{-1} \cdot \sqrt{15}\sqrt{-1}$

$i\sqrt{6} \cdot i\sqrt{15}$

$= i^2\sqrt{6 \cdot 15} = -1\sqrt{90}$

$= -1\sqrt{9 \cdot 10} = -3\sqrt{10}$

4) i^{31}

$= i^{30} \cdot i$

$(i^2)^{15} \cdot i$

$= (-1)^{15} \cdot i$

$= -1i$

5) $3i \cdot 4i$

$3 \cdot 4 \cdot i \cdot i$

$= 12i^2$

$= 12(-1)$

$= -12$

6) $\sqrt{-20} \cdot \sqrt{-12}$

$\sqrt{20}\sqrt{-1} \cdot \sqrt{12}\sqrt{-1}$

$i^2\sqrt{240}$

$= -1\sqrt{16 \cdot 15}$

$= -4\sqrt{15}$

7) $3\sqrt{-24} \cdot 2\sqrt{-18}$

$3\sqrt{24}\sqrt{-1} \cdot 2\sqrt{18}\sqrt{-1}$

$3i \cdot 2i\sqrt{24 \cdot 18}$

$6i^2\sqrt{432}$

$-6\sqrt{144 \cdot 3}$

$-6 \cdot 12\sqrt{3} = -72\sqrt{3}$

Solve the following equations:

Ex. $x^2 = -144$

$x = \pm\sqrt{144}\sqrt{-1}$

$x = \pm 12i$

Ex. $x^2 + 48 = 0$

$x^2 = -48$

$x = \pm\sqrt{16 \cdot 3}\sqrt{-1}$

$= \pm 4i\sqrt{3}$

Ex. $25x^2 + 9 = 0$

$25x^2 = -9$

$x^2 = \frac{\pm 9}{25} = \pm\sqrt{\frac{9}{25}}\sqrt{-1}$

$x = \pm\frac{3}{5}i$

8) $9x^2 + 25 = 0$

$9x^2 = -25$

$x^2 = \frac{-25}{9}$

$x = \pm\sqrt{\frac{25}{9}}\sqrt{-1}$

$= \pm\frac{5}{3}i$

9) $4x^2 + 100 = 0$

$\frac{4x^2}{4} = \frac{-100}{4}$

$x^2 = -25$

$x = \pm\sqrt{-25}$

$= \pm\sqrt{25}\sqrt{-1}$

$= \pm 5i$

10) $x^2 + 64 = 0$

$x^2 = -64$

$x = \pm\sqrt{64}\sqrt{-1}$

$= \pm 8i$