Key

4.4 Notes Day 2

Objectives: Understand what complex numbers are, Solve equations with Complex Numbers, Add/Subtract Complex Number, and Multiply/Divide Complex Numbers

A **complex number** is a number that can be written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. The set of real numbers is a subset of the set of complex numbers C.

Every complex number has a **real part** a and an **imaginary part** b.

Real part Imaginary part

$$\underset{\text{Real part}}{a} + \underset{\text{Imaginary part}}{b}i$$

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.

Ex. Find the values of x and y that make the equation true $4x + 10i = 2 - (4y)i$

$\frac{4x}{4} = \frac{2}{4}$ $x = \frac{1}{2}$ $10i = -4yi$ $\frac{10}{-4} = \frac{-4y}{-4}$ $y = \frac{-5}{2}$

1) **Find the values of x and y that make each equation true.** $2x - 6i = -8 + (20y)i$

$\frac{2x}{2} = \frac{-8}{2}$ $x = -4$ $-6i = (20y)i$ $\frac{-6}{20} = \frac{20y}{20}$ $\frac{-3}{10} = y$

2) **Find the values of x and y that make each equation true.** $-8 + (6y)i = 5x - i\sqrt{6}$

$\frac{-8}{5} = \frac{5x}{5}$ $x = \frac{-8}{5}$ $\frac{(6y)i}{6} = \frac{-i\sqrt{6}}{6}$ $y = \frac{\sqrt{6}}{6}$

Adding and Subtracting Complex Numbers: To add or subtract complex numbers, combine like terms.

That is, combine the real parts, and combine the imaginary parts.

Ex. $(4 - i) - (5 + 8i)$

$4 - i - 5 - 8i = -1 - 9i$

Ex. $6i + (3 - 4i)$

$6i + 3 - 4i = 3 + 2i$

3) $(6 - 2i) + (-6 + 2i)$

$6 - 2i + (-6) + 2i$

0

4) $(4 - 8i) - (3 - 6i)$

$4 - 8i - 3 + 6i$

$= 1 - 2i$

5) $(5 - 7i) + (2 + 4i)$

$5 - 7i + 2 + 4i$

$7 - 3i$

Multiplying Complex Numbers: To multiply complex numbers use the FOIL or Double Distribution method of multiplying binomials.

Ex. $-2i(2-4i)$

$-4i+8i^2$

$=-4i+8(-1) = -8-4i$

Ex. $(3+6i)(4-i)$

$12-3i+24i-6i^2$

$12+21i-6(-1) = 18+21i$

Ex. $(2+9i)(2-9i)$

$4\underbrace{-18i+18i}-81i^2$

$=4-81(-1)=4+81=85$

6) $(3+5i)(5-3i)$

$15\underbrace{-9i+25i}-15i^2$

$15+16i-15(-1)$

$=30+16i$

7) $(4-i)(6-6i)$

$24\underbrace{-24i-6i}+6i^2$

$24-30i+6(-1)$

$=18-30i$

8) $(-6-i)(3-3i)$

$-18\underbrace{+18i-3i}+3i^2$

$-18+15i+3(-1)$

$=-21+15i$

The solutions $-5+i\sqrt{10}$ and $-5-i\sqrt{10}$ are related. These solutions are a *complex conjugate* pair. Their real parts are equal and their imaginary parts are opposites. The complex conjugate of any complex number $a+bi$ is the complex number $a-bi$.

Find each complex conjugate.

Ex. 8+5i

$8-5i$

9) 9-i

$9+i$

10) $i+\sqrt{3}$

Step1: $\sqrt{3}+i$

Step2: $\sqrt{3}-i$

$a+bi$ form

Dividing Complex Numbers: The product of complex conjugates is always a real number. You can use this fact to simplify the quotient of two complex numbers. Multiply the numerator and denominator by the complex conjugate of the denominator and then simplify.

Ex. $\frac{1+8i}{4-2i}\cdot\frac{(4+2i)}{(4+2i)} = \frac{4+2i+32i+16i^2}{16+\cancel{8i}-\cancel{8i}-4i^2}$

$\frac{4+34i+16(-1)}{16-4(-1)} = \frac{-12+34i}{20} = \frac{-12}{20}+\frac{34}{20}i$

$=\frac{-3}{5}+\frac{17}{10}i$

Ex. $\frac{4+i}{5i}\cdot\frac{(-5i)}{(-5i)} = \frac{-20i-5i^2}{-25i^2}$

$=\frac{-20i-5(-1)}{-25(-1)}$

$=\frac{-20i}{+25}+\frac{5}{25}$

$=\frac{1}{5}-\frac{4}{5}i$

11) $\frac{-2i}{3+5i}\cdot\frac{(3-5i)}{(3-5i)}$

$\frac{-6i+10i^2}{9-\cancel{15i}+\cancel{15i}-25i^2}$

$=\frac{-6i+10(-1)}{9-25(-1)} = \frac{-10-6i}{34} = \frac{-5}{17}-\frac{3}{17}i$

12) $\frac{2i}{1+i}\cdot\frac{(1-i)}{(1-i)}$

$\frac{2i-2i^2}{1-\cancel{i}+\cancel{i}-i^2}$

$=\frac{2i-2(-1)}{1-(-1)} = \frac{2+2i}{2} = 1+i$