Key

4.5 Day 2 Notes

Objectives: Solve Quadratic Equations by Completing the Square

Solve each equation by completing the square.

1) $x^2 + 10x + 9 = 0$

$\left(\frac{10}{2}\right)^2 = 25$

$x^2 + 10x + \underline{25} = -9 + \underline{25}$

$(x+5)^2 = 16$

$\sqrt{(x+5)^2} = \pm\sqrt{16}$

$x + 5 = \pm 4$

$-5 \quad -5$

$x = -5 + 4$ or $x = -5 - 4$

$x = -1$ $\quad x = -9$

2) $x^2 + 6x + 8 = 0$

$\left(\frac{6}{2}\right)^2 = 9$

$x^2 + 6x + \underline{9} = -8 + \underline{9}$

$(x+3)^2 = 1$

$\sqrt{(x+3)^2} = \pm\sqrt{1}$

$x + 3 = \pm 1$

$-3 \quad -3$

$x = -3 + 1$ or $x = -3 - 1$

$x = -2$ or $x = -4$

3) $x^2 - 2x - 3 = 0$

$\left(\frac{2}{2}\right)^2 = 1$

$x^2 - 2x + \underline{1} = 3 + \underline{1}$

$(x-1)^2 = 4$

$\sqrt{(x-1)^2} = \pm\sqrt{4}$

$x - 1 = \pm 2$

$+1 \quad +1$

$x = -1 + 2$ or $x = -1 - 2$

$x = 1$ $\quad x = -3$

4) $x^2 + 32x + 255 = 0$

$x^2 + 32x + \underline{256} = -255 + \underline{256}$ $\quad \left(\frac{32}{2}\right)^2 = 256$

$(x+16)^2 = 1$

$\sqrt{(x+16)^2} = \pm\sqrt{1}$

$x + 16 = \pm 1$

$-16 \quad -16$

$x = -16 + 1$ or $x = -16 - 1$

$x = -15$ $\quad x = -17$

Not all solutions of quadratic equations are real numbers. In some cases, the solutions are complex number of the form $a + bi$, where $b \neq 0$.

Ex. $x^2 + 8x + 22 = 0$

$\left(\frac{8}{2}\right)^2 = 16$

$x^2 + 8x + \underline{16} = -22 + \underline{16}$

$(x+4)^2 = -6$

$\sqrt{(x+4)^2} = \pm\sqrt{-6}$

$x + 4 = \pm i\sqrt{6}$

$-4 \quad -4$

$x = -4 + i$ or $x = -4 - i$

Ex. $x^2 + 2x + 2 = 0$

$\left(\frac{2}{2}\right)^2 = 1$

$x^2 + 2x + \underline{1} = -2 + \underline{1}$

$(x+1)^2 = -1$

$\sqrt{(x+1)^2} = \pm\sqrt{-1}$

$x + 1 = \pm i$

$-1 \quad -1$

$x = -1 + i$ or $x = -1 - i$

5) $x^2 - 6x + 25 = 0$

$\left(\frac{-6}{2}\right)^2 = 9$

$x^2 - 6x + \underline{9} = -25 + \underline{9}$

$(x-3)^2 = -16$

$\sqrt{(x-3)^2} = \pm\sqrt{-16}$

$x - 3 = \pm 4i$

$+3 \quad +3$

$x = -3 + 4i$ or $x = -3 - 4i$

6) $x^2 + 4x + 11 = 0$

$\left(\frac{4}{2}\right)^2 = 4$

$x^2 + 4x + \underline{4} = -11 + \underline{4}$

$(x+2)^2 = -7$

$\sqrt{(x+2)^2} = \pm\sqrt{-7}$

$x + 2 = \pm i\sqrt{7}$

$-2 \quad -2$

$x = -2 + i\sqrt{7}$ or $x = -2 - i\sqrt{7}$

When the coefficient of the quadratic term is not 1, you must divide the equation by that coefficient before completing the square and solving the problem.

Ex. $\frac{2x^2}{2} - \frac{12x}{2} + \frac{8}{2} = 0$

$x^2 - 6x + 4 = 0$

$\left(\frac{6}{2}\right)^2 = 9$

$x^2 - 6x + \underline{9} = -4 + \underline{9}$

$(x-3)^2 = 5$

$\sqrt{(x-3)^2} = \pm\sqrt{5}$

$x - 3 = \pm\sqrt{5}$

$+3 \quad +3$

$x = 3 + \sqrt{5}$ or $x = 3 - \sqrt{5}$

Ex. $\frac{2x^2}{2} - \frac{3x}{2} - \frac{5}{2} = 0$

$\left(\frac{\frac{3}{2}}{2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$x^2 - \frac{3}{2}x - \frac{5}{2} = 0$

$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{5}{2} + \frac{9}{16}$

$\left(x - \frac{3}{4}\right)^2 = \frac{40}{16} + \frac{9}{16}$

$\sqrt{(x-3/4)^2} = \pm\frac{\sqrt{49}}{\sqrt{16}}$

$x - 3/4 = \pm\frac{7}{4} + 3/4$

$+3/4$

$x = \frac{10}{4}$ or $x = -1$

7) $\frac{4x^2}{4} + \frac{32x}{4} + \frac{16}{4} = 0$

$x^2 + 8x + 4 = 0$

$\left(\frac{8}{2}\right)^2 = 16$

$x^2 + 8x + \underline{16} = -4 + \underline{16}$

$\sqrt{(x+4)^2} = \sqrt{12}$

$x + 4 = \pm\sqrt{4 \cdot 3}$

$x + 4 = \pm 2\sqrt{3}$

$-4 \quad -4$

$x = -4 + 2\sqrt{3}$

or $x = -4 - 2\sqrt{3}$

8) $\frac{2x^2}{2} + \frac{8x}{2} + \frac{5}{2} = 0$

$\left(\frac{4}{2}\right)^2 = 4$

$x^2 + 4x + \underline{4} = -\frac{5}{2} + \underline{4}$

$(x+2)^2 = -\frac{5}{2} + \frac{8}{2}$

$\sqrt{(x+2)^2} = \pm\sqrt{\frac{3}{2}}$

$x + 2 = \pm\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x + 2 = \pm\frac{\sqrt{6}}{2}$

$-2 \quad -2$

$x = -2 + \frac{\sqrt{6}}{2}$ or $x = -2 - \frac{\sqrt{6}}{2}$