Completed

Investigating Monomials (pp. 1 of 4)

Definition

1) **Monomial**—What is a *monomial*?

The following are monomials:

2	$\frac{5}{3}$	-8.5
$2x$	$\frac{5}{3}y^2$	$-8.5xy^2$
0	$9a^2b^3c^7$	$\sqrt{5}$

So monomials can include:

- real numbers
- variables
- exponents - whole #s

However, these are not monomials:

$2+x$	$x^2-8.5$
$\frac{9}{x}$	$\sqrt{x}$

So a monomial cannot have:

- addition
- subtraction
- variable in a radical $\sqrt{\ }$
- variable in denom

Vocabulary

Using the monomials above, identify examples of the following definitions.

2) A constant is a monomial with no variables,

Examples: 2, $\frac{5}{3}$, -8.5, 0, $\sqrt{5}$

3) In a monomial, the leading coefficient is the numeric factor of the variable (or variables)

Examples: $2x$ $\frac{5}{3}y^2$ $-8.5xy^2$ $9a^2b^3c^7$

4) The degree of a monomial is the sum of the exponents on the variables only

Examples:

Monomial	
$2x$	1
$\frac{5}{3}y^2$	2
$-8.5xy^2$	3

Monomial	
$9a^2b^3c^7$	12
$\sqrt{5}$	0
0	0

Investigating Monomials (pp. 2 of 4)

Simplifying Monomials

Multiplying	Dividing
Sample: $(5x^2y^3)(4xy^4)$ Expand: $5 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot 4 \cdot x \cdot y \cdot y \cdot y \cdot y$ Re-order: $5 \cdot 4 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$ Simplify: $\boxed{20x^3y^7}$	Sample: $\frac{18x^4y^3}{6xy^2}$ Expand: $\frac{18 \cdot \cancel{x} \cdot x \cdot x \cdot x \cdot \cancel{y} \cdot \cancel{y} \cdot y}{6 \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{y}}$ Simplify: $\boxed{3x^3y}$
• What's the short-cut? multiply the coefficients add the exponents	• What's the short-cut? divide the coefficients subtract the exponents
• What's the rule? $a^m \cdot a^n = \boxed{a^{m+n}}$ Note: Bases must be the same.	• What's the rule? $\frac{a^m}{a^n} = \boxed{a^{m-n}}$ Note: Bases must be the same.

Other Rules	Samples		
$a^{-n} = \frac{1}{a^n}$	$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$	$x^{-4} = \frac{1}{x^4}$	$\frac{b^3}{b^9} = b^{3-9} = b^{-6} = \frac{1}{b^6}$
$(a^m)^n = a^{mn}$	$(5^2)^3 = 5^{2 \cdot 3} = 5^6$ $= 15625$	$(x^4)^5 = x^{4 \cdot 5} = x^{20}$	$(b^{-2})^6 = b^{-2 \cdot 6} = b^{-12} = \frac{1}{b^{12}}$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{8}\right)^2 = \frac{5^2}{8^2} = \frac{25}{64}$	$\left(\frac{x}{y}\right)^7 = \frac{x^7}{y^7}$	$\left(\frac{b^3}{c^2}\right)^5 = \frac{b^{3 \cdot 5}}{c^{2 \cdot 5}} = \frac{b^{15}}{c^{10}}$
$(ab)^m = a^m b^m$	$(5x^2)^2 = 5^2(x^2)^2$ $= 25x^{2 \cdot 2}$ $= 25x^4$	$(xy^2)^6 = x^6(y^2)^6$ $x^6y^{2 \cdot 6}$ x^6y^{12}	$(4x^2y^4)^3 = 4^3(x^2)^3(y^4)^3$ $= 64x^{2 \cdot 3}y^{4 \cdot 3}$ $= 64x^6y^{12}$

Investigating Monomials (pp. 3 of 4)

Sample Problems

1) $\left(2ab^2\right)\left(-4a^3b^3c\right) = 2\cdot -4\ a^{1+3}\ b^{2+3}\ c$

$\boxed{= -8a^4b^5c}$

2) $\left(3x^2\right)^3 = 3^3\left(x^2\right)^3$

$= 27x^{2\cdot 3} = \boxed{27x^6}$

3) $\dfrac{x^{11}}{x^5} = x^{11-5} = \boxed{x^6}$

4) $\left(6x^2y^3\right)\left(xyz\right)^3 = \left(6x^2y^3\right)\left(x^3y^3z^3\right)$

$= 6x^{2+3}\ y^{3+3}\ z^3$

$\boxed{= 6x^5y^6z^3}$

5) $\left(6a^{-3}b^2\right)^{-3} = \dfrac{1}{\left(6a^{-3}b^2\right)^3} =$

$= \dfrac{1}{6^3\left(a^{-3}\right)^3\left(b^2\right)^3} = \dfrac{1}{216a^{-9}b^6} = \boxed{\dfrac{a^9}{216b^6}}$

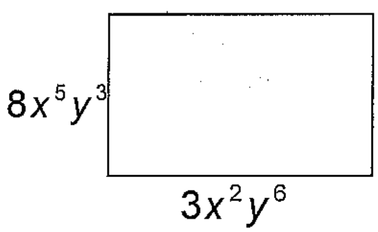
6) $\left(\dfrac{-3x^3y^6}{x^5y^{-2}z^{-1}}\right)^2 = \dfrac{(-3)^2\left(x^3\right)^2\left(y^6\right)^2}{\left(x^5\right)^2\left(y^{-2}\right)^2\left(z^{-1}\right)^2}$

$= \dfrac{9x^6y^{12}}{x^{10}y^{-4}z^{-2}} = 9x^{6-10}\ y^{12--4}\ z^{0--2}$

$= 9x^{-4}y^{16}z^2 = \boxed{\dfrac{9y^{16}z^2}{x^4}}$

7) A rectangle has a width represented by $3x^2y^6$ and a length represented by $8x^5y^3$. What expression can be used to represent the area of the rectangle?

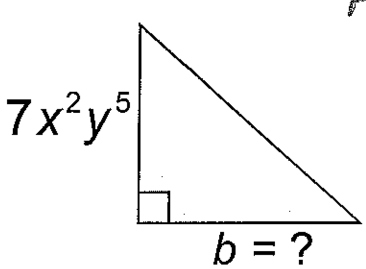
$l \cdot w$



$\left(8x^5y^3\right)\cdot\left(3x^2y^6\right)$

$= 8\cdot 3\ x^{5+2}\ y^{3+6} = \boxed{24x^7y^9}$

8) The area of the triangle below is represented by $14x^4y^9$. Find the expression that represents the base of the triangle. $A = \frac{1}{2}b\cdot h$



$14x^4y^9 = \frac{1}{2}b\cdot 7x^2y^5$

$2\left(14x^4y^9\right) = 2\left(\frac{1}{2}b\cdot 7x^2y^5\right)$

$28x^4y^9 = b\cdot 7x^2y^5$

$\dfrac{28x^4y^9}{7x^2y^5} = b\cdot\dfrac{\cancel{7x^2y^5}}{\cancel{7x^2y^5}} \longrightarrow b = \dfrac{28x^4y^9}{7x^2y^5}$

$= 4x^{4-2}y^{9-5}$

$\boxed{= 4x^2y^4}$

Key

Investigating Monomials (pp. 4 of 4)

HW

Practice Problems Simplify the following expressions.

1) $x^3 \bullet x^3 \bullet x \bullet x^5$

$x^{3+3+1+5} = \boxed{x^{12}}$

2) $(-3xy^2)^3 = (-3)^3(x)^3(y^2)^3$

$\boxed{= -27x^3y^6}$

3) $\dfrac{-x^6y^6}{x^3y^4} = -1 \cdot x^{6-3}y^{6-4}$

$\boxed{= -x^3y^2}$

4) $(3x^2y^6)(-4x^2y^6)$

$3 \cdot -4\, x^{2+2}\, y^{6+6} = \boxed{-12x^4y^{12}}$

5) $\dfrac{ab^4c^6}{a^5bc^2} = a^{1-5}b^{4-1}c^{6-2}$

$= a^{-4}b^3c^4 = \boxed{\dfrac{b^3c^4}{a^4}}$

6) $\dfrac{2a^5b^3c^3}{8a^3b^3c} = \dfrac{1}{4}a^{5-3}b^{3-3}c^{3-1} = \dfrac{1}{4}a^2b^0c^2$

$\boxed{= \dfrac{a^2c^2}{4}}$

7) $\dfrac{40a^{-1}b^{-7}}{20a^{-5}b^3} = 2a^{-1+5}b^{-7-3}$

$= 2a^4b^{-10} = \boxed{\dfrac{2a^4}{b^{10}}}$

8) $\dfrac{(-15m^5n^8)(m^3n^2)^{-2}}{45m^4n} = \dfrac{-15m^5n^8}{(45m^4n)(m^3n^2)^2}$

$= \dfrac{-15m^5n^8}{(45m^4n)(m^6n^4)} = \dfrac{-15m^5n^8}{45m^{10}n^5} = \boxed{\dfrac{-n^3}{3m^5}}$

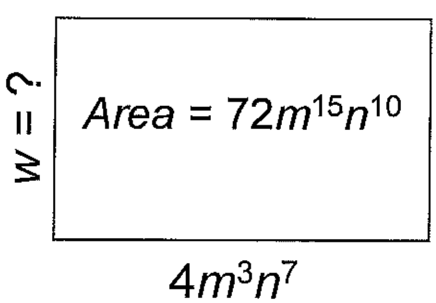
9) The height of a triangle is represented by the expression $15p^6qr^3$. The base is represented by $8p^2q^3r^5$. Find the expression that can be used to represent the area of the triangle.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(8p^2q^3r^5)(15p^6qr^3)$

$A = 60p^8q^4r^8$

10) The length and area of a rectangle are given in the diagram below. Find the expression that can be used to represent the width of the rectangle.



$A = lw$

$72m^{15}n^{10} = (4m^3n^7)w$

$\dfrac{72m^{15}n^{10}}{4m^3n^7} = w$

$\boxed{18m^{12}n^3 = w}$