Key

5.2 – Polynomials & Long Division

Just as we performed long division with numbers, we can also perform long division with polynomials.

A) Dividing a polynomial by a monomial.

Ex: Simplify. $\frac{6x^4y^3 + 12x^3y^2 - 18x^2y}{3xy}$

$$\frac{6x^4y^3}{3xy} + \frac{12x^3y^2}{3xy} - \frac{18x^2y}{3xy}$$

$$= 2x^3y^2 + 4x^2y - 6x$$

1. $\frac{20c^4d^2f - 16cdf^2 + 4cdf}{4cdf}$

$$\frac{20c^4d^2f}{4cdf} - \frac{16cdf^2}{4cdf} + \frac{4cdf}{4cdf}$$

$$= 5c^3d - 4f + 1$$

B) Dividing a polynomial by a binomial.

Ex: $(2x^3 + 3x^2 - 4x + 15) \div (x + 3)$

$$
\begin{array}{r}
2x^2 - 3x + 5 \\
x+3 \;\big)\; 2x^3 + 3x^2 - 4x + 15 \\
-\underline{(2x^3 + 6x^2)} \qquad\qquad \\
-3x^2 - 4x \qquad \\
-\underline{(-3x^2 - 9x)} \qquad \\
5x + 15 \\
-\underline{(5x + 15)} \\
0
\end{array}
$$

1. $(3x^3 - 8x^2 + 11x - 14) \div (x - 2)$

$$
\begin{array}{r}
3x^2 - 2x + 7 \\
x-2 \;\big)\; 3x^3 - 8x^2 + 11x - 14 \\
-\underline{(3x^3 - 6x^2)} \qquad\qquad \\
-2x^2 + 11x \qquad \\
-\underline{(-2x^2 + 4x)} \qquad \\
7x - 14 \\
-\underline{(7x - 14)} \\
0
\end{array}
$$

Ex. $(15x^2 + 8x + 1) \div (3x + 1)$

$$
\begin{array}{r}
5x + 1 \\
3x+1 \;\big)\; 15x^2 + 8x + 1 \\
-\underline{(15x^2 + 5x)} \qquad \\
3x + 1 \\
-\underline{(3x + 1)} \\
0
\end{array}
$$

Check:

$(3x+1)(5x+1) = 15x^2 + 3x + 5x + 1$

✓ $= 15x^2 + 8x + 1$

2. $(2x^2 + 3x - 20) \div (2x - 5)$

$$
\begin{array}{r}
x + 4 \\
2x-5 \;\big)\; 2x^2 + 3x - 20 \\
-\underline{(2x^2 - 5x)} \qquad \\
8x - 20 \\
-\underline{(8x - 20)} \\
0
\end{array}
$$

Check: $(2x-5)(x+4)$

$= 2x^2 + 8x - 5x - 20$

✓ $2x^2 + 3x - 20$

C) Inserting zero-place holders for any missing exponents.

Ex: $(2x^3 - 3x^2 - 80) \div (x - 4)$

$$\begin{array}{r} 2x^2 + 5x + 20 \\ x - 4 \,\big)\, \overline{2x^3 - 3x^2 + 0x - 80} \\ -(2x^3 - 8x^2) \\ \hline 5x^2 + 0x \\ -(5x^2 - 20x) \\ \hline 20x - 80 \\ -(20x - 80) \\ \hline 0 \end{array}$$

3. $(3x^3 - 14x - 39) \div (x - 3)$

$$\begin{array}{r} 3x^2 + 9x + 13 \\ x - 3 \,\big)\, \overline{3x^3 + 0x^2 - 14x - 39} \\ -(3x^3 - 9x^2) \\ \hline 9x^2 - 14x \\ -(9x^2 - 27x) \\ \hline 13x - 39 \\ -(13x - 39) \\ \hline 0 \end{array}$$

D) Remainders

Ex. $(15x^2 + 8x + 12) \div (3x + 1)$

$$\begin{array}{r} 5x + 1 \\ 3x + 1 \,\big)\, \overline{15x^2 + 8x + 12} \\ -(15x^2 + 5x) \\ \hline 3x + 12 \\ -(3x + 3) \\ \hline 9 \end{array}$$

Quotient answer: $5x + 1 + \frac{9}{3x+1}$

watch out! 4. $(a^3 + 2a^2 - 9) \div (a - 2)$

$$\begin{array}{r} a^2 + 4a + 8 \\ a - 2 \,\big)\, \overline{a^3 + 2a^2 + 0a - 9} \\ -(a^3 - 2a^2) \\ \hline 4a^2 + 0a \\ -(4a^2 - 8a) \\ \hline 8a - 9 \\ -(8a - 16) \\ \hline 7 \end{array}$$

Quotient: $a^2 + 4a + 8 + \frac{7}{a-2}$

E) Divisor with a degree greater than one.

Ex: $(r^3 + 4r^2 - r - 9) \div (r^2 - 1)$

$$\begin{array}{r} r + 4 \\ r^2 - 1 \,\big)\, \overline{r^3 + 4r^2 - r - 9} \\ -(r^3 \qquad - r) \\ \hline 4r^2 + 0r - 9 \\ -(4r^2 \qquad - 4) \\ \hline 5 \end{array}$$

Quotient: $r + 4 + 5/r^2 - 1$

5. $(x^3 - 3x^2 + x + 7) \div (x^2 + 1)$

$$\begin{array}{r} x - 3 \\ x^2 + 1 \,\big)\, \overline{x^3 - 3x^2 + x + 7} \\ -(x^3 \qquad + x) \\ \hline -3x^2 \qquad + 7 \\ -(-3x^2 \qquad - 3) \\ \hline 10 \end{array}$$

Quotient: $x - 3 + \frac{10}{x^2+1}$

Long Division is not limited to dividing only by a binomial.

Ex. $(x^3 - 2x^2 - 9x + 18) \div (x^2 - 5x + 6)$

$$\begin{array}{r} x + 3 \\ x^2 - 5x + 6 \,\big)\, \overline{x^3 - 2x^2 - 9x + 18} \\ -(x^3 - 5x^2 + 6x) \\ \hline 3x^2 - 15x + 18 \\ -(3x^2 - 15x + 18) \\ \hline 0 \end{array}$$

Check: $(x+3)(x^2 - 5x + 6)$

$$\begin{array}{r} = x^3 - 5x^2 + 6x \qquad\quad \\ + \quad 3x^2 - 15x + 18 \\ \hline x^3 - 2x^2 - 9x + 18 \end{array}$$ ✓