Key

5.2 Day 2 – Polynomials & Synthetic Division

Warm-Up: Find the quotient using long division.

$(x^4 + x^3 - 6x^2 - 4x + 14) \div (x^2 + x - 2)$

$$
\begin{array}{r}
x^2 - 4 \\
x^2+x-2 \,\overline{)\, x^4+x^3-6x^2-4x+14} \\
-(x^4+x^3-2x^2) \\
\hline
-4x^2-4x+14 \\
-(-4x^2-4x+8) \\
\hline
6
\end{array}
$$

Quotient: $x^2 - 4 + \frac{6}{x^2+x-2}$

Synthetic division is an efficient way to divide a polynomial by a linear binomial using only coefficients.

To use synthetic division:

- The polynomial must be in standard form (exponents in descending order)
- Must include 0 for any skipped ~~exponents~~ terms
- Synthetic division only works when dividing by a linear binomial.

Let's compare long division and synthetic division. **Ex:** $(2y^3 - y^2 + 25) \div (y - 3)$

$y - 3 = 0$, $+3\ +3$, $y = 3$

Long division:

$$
\begin{array}{r}
2y^2+5y+15 \\
y-3\,\overline{)\,2y^3 - y^2 + 0y + 25} \\
-(2y^3-6y^2) \\
\hline
5y^2+0y \\
-(5y^2-15y) \\
\hline
15y+25 \\
-(15y-45) \\
\hline
\boxed{70}
\end{array}
$$

synthetic division:

$$
\begin{array}{r|rrrr}
3 & 2 & -1 & 0 & 25 \\
(+) & \downarrow & 6 & 15 & 45 \\
\hline
\text{multiply} & 2 & 5 & 15 & \boxed{70}
\end{array}
$$

← box last # = remainder!

Quotient: $2y^2 + 5y + 15 + \frac{70}{y-3}$

Use synthetic division to find each quotient.

Ex. $(x^3 - 7x^2 + 15x - 9) \div (x - 3)$

$$
\begin{array}{r|rrrr}
3 & 1 & -7 & 15 & -9 \\
+ & \downarrow & 3 & -12 & 9 \\
\hline
\text{multiply} & 1 & -4 & 3 & \boxed{0}
\end{array}
$$

Quotient $= x^2 - 4x + 3$

Check: $(x-3)(x^2-4x+3)$

$= x^3 - 4x^2 + 3x$
$\quad - 3x^2 + 12x - 9$

$\rightarrow x^3 - 7x^2 + 15x - 9$ ✓

1. $(x^3 - 3x^2 - 5x - 25) \div (x - 5)$

$$
\begin{array}{r|rrrr}
5 & 1 & -3 & -5 & -25 \\
+ & \downarrow & 5 & 10 & 25 \\
\hline
 & 1 & 2 & 5 & \boxed{0}
\end{array}
$$

$x^2 + 2x + 5$ = Quotient

Check: $(x-5)(x^2+2x+5)$

$= x^3 + 2x^2 + 5x$
$+ \quad -5x^2 - 10x - 25$

$x^3 - 3x^2 - 5x - 25$ ✓

2. $(g^3 - 4g^2 - 8g - 3) \div (g + 1)$

$$
\begin{array}{r|rrrr}
-1 & 1 & -4 & -8 & -3 \\
+ & \downarrow & -1 & 5 & 3 \\
\hline
\text{multiply} & 1 & -5 & -3 & \boxed{0}
\end{array}
$$

$g^2 - 5g - 3$ = Quotient

3. $(3x^3 + 17x^2 + 21x - 9) \div (x + 3)$

$$
\begin{array}{r|rrrr}
-3 & 3 & 17 & 21 & -9 \\
+ & \downarrow & -9 & -24 & 9 \\
\hline
 & 3 & 8 & -3 & \boxed{0}
\end{array}
$$

Quotient: $3x^2 + 8x - 3$

Watch out for missing terms!

Ex. $(y^3 + 4y^2 + 4y) \div (y + 2)$

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 4 & 0 \\ + & \downarrow & -2 & -4 & 0 \\ \hline & 1 & 2 & 0 & \boxed{0} \end{array}$$

Quotient: $y^2 + 2y$

Check: $(y+2)(y^2+2y)$

$= y^3 + 2y^2 + 2y^2 + 4y$ $\boxed{y^3+4y^2+4y}$ ✓

4. $(2b^4 + 6b^3 + 5b^2 - 45) \div (b + 3)$

$$\begin{array}{r|rrrrr} -3 & 2 & 6 & 5 & 0 & -45 \\ + & \downarrow & -6 & 0 & -15 & 45 \\ \hline & 2 & 0 & 5 & -15 & \boxed{0} \end{array}$$

$2b^3 + 0b^2 + 5b - 15$

or $2b^3 + 5b - 15$

Practicing with remainders.

Ex. $(x^3 + 3x^2 - x + 2) \div (x - 1)$

$$\begin{array}{r|rrrr} 1 & 1 & 3 & -1 & 2 \\ + & \downarrow & 1 & 4 & 3 \\ \hline & 1 & 4 & 3 & \boxed{5} \end{array}$$

Quotient: $x^2 + 4x + 3 + \frac{5}{x-1}$

5. $(9x^3 - 48x^2 + 13x + 3) \div (x - 5)$

$$\begin{array}{r|rrrr} 5 & 9 & -48 & 13 & 3 \\ + & \downarrow & 45 & -15 & -10 \\ \hline & 9 & -3 & -2 & \boxed{-7} \end{array}$$

$\boxed{9x^2 - 3x - 2 - \frac{7}{x-5}}$

Quotient

What if the divisor has a coefficient in front of the variable?

Ex. $(2x^3 + 9x^2 + 14x + 5) \div (2x + 1)$

$2x+1=0$
$2x=-1$
$x=-\frac{1}{2}$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 9 & 14 & 5 \\ + & \downarrow & -1 & -4 & -5 \\ \hline & 2 & 8 & 10 & \boxed{0} \end{array}$$

Quotient $= \frac{2x^2}{2} + \frac{8x}{2} + \frac{10}{2}$

Check: $(2x+1)(2x^2+8x+10)$

Doesn't work!

$4x^3 + 16x^2 + 20x$
$+ 2x^2 + 8x + 10$

$4x^3 + 18x^2 + 28x + 10$

B/c divided out "2" from divisor, must divide out 2 from quotient: $\boxed{(2x+1)(x^2+4x+5)}$

6. $(9x^3 - 18x^2 - x + 2) \div (3x + 1)$

$3x+1=0$
$3x=-1$
$x=-1/3$

$$\begin{array}{r|rrrr} -1/3 & 9 & -18 & -1 & 2 \\ + & \downarrow & -3 & 7 & -2 \\ \hline & 9 & -21 & 6 & \boxed{0} \end{array}$$

$\frac{9x^2}{3} - \frac{21x}{3} + \frac{6}{3}$

Check

$\boxed{(3x+1)(3x^2-7x+2)}$

$9x^3 - 21x^2 + 6x$
$+3x^2 - 7x + 2$

$9x^3 - 18x^2 - x + 2$ ✓