5.3 notes
Polynomial Functions

Key

Objectives: 1. Evaluate polynomial functions
2. Identify general shapes of graphed polynomial functions

Standard form: A polynomial with one variable is written in standard form when exponents are in descending order.

Degree: Is the single highest degree of any one term.

Leading coefficient: The coefficient on the first term when written in standard form.

Practice: Write in standard form & identify degree and leading coefficient.

Ex. $2x^2 + 8x^5 - 4x^3 - x - 3$

Standard form: $8x^5 - 4x^3 + 2x^2 - x - 3$

Degree: 5

Leading coefficient: 8

1. $3x^4 + 6x^3 - 4x^8 + 2x$

standard form: $-4x^8 + 3x^4 + 6x^3 + 2x$

degree: 8

Leading coefficient: 4

2. $-x^3 - 4x^2 + 2x$

Standard form: $-x^3 - 4x^2 + 2x$

Degree: 3

Leading coefficient: -1

3. $7x^3 - 5x + x^6 + 1$

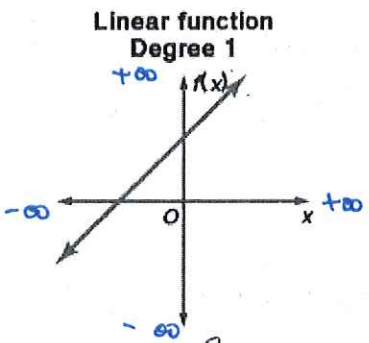
standard form: $x^6 + 7x^3 - 5x + 1$

degree: 6

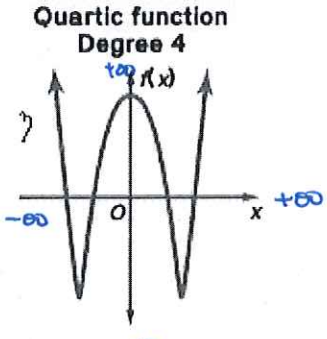
Leading coefficient: 1

Polynomial Functions are even or odd, depending on their degree! Odd degree functions have ending behavior similar to a line. Even degree functions relate to a parabola.

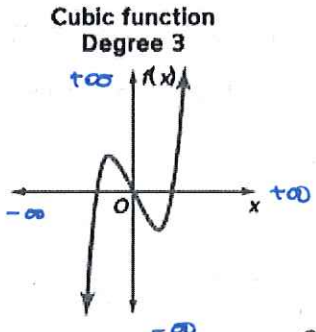
Ex.



$x \to +\infty,\ f(x) \to +\infty$
$x \to -\infty,\ f(x) \to -\infty$



$x \to \pm\infty,\ f(x) \to +\infty$

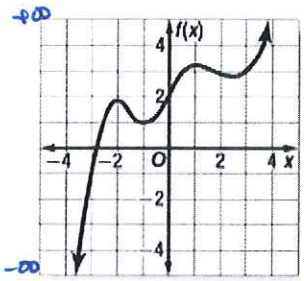


$x \to +\infty,\ f(x) \to +\infty$
$x \to -\infty,\ f(x) \to -\infty$

Ending behavior: is described as x approaches positive infinity ($x \to +\infty$) or negative infinity ($x \to -\infty$) and describes the increase or decrease in the y values ($f(x) \to \pm\infty$)

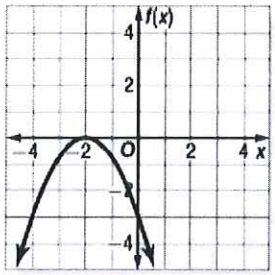
Practice:
For each graph, state whether it represents an odd or even degree function, state the number of real zeros, and describe its ending behavior.

1.



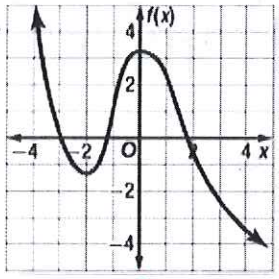
$x \to \infty, f(x) \to +\infty$
$x \to -\infty, f(x) \to -\infty$

2.



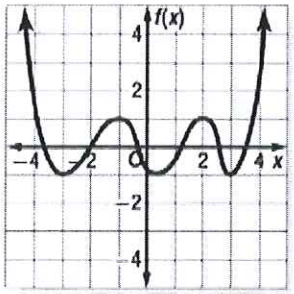
$x \to \pm\infty, f(x) \to -\infty$

3.



$x \to +\infty, f(x) \to -\infty$
$x \to -\infty, f(x) \to +\infty$

4.



$x \to \pm\infty, f(x) \to +\infty$

Evaluating polynomial functions for numbers, variables and algebraic expressions.

Ex. If $f(x) = x^2 + 2x - 3$,

ex. Find: $f(-5)$

$f(-5) = (-5)^2 + 2(-5) - 3$
$= 25 - 10 - 3 = \boxed{12}$

1. Find : $f(-1)$

$f(-1) = (-1)^2 + 2(-1) - 3$
$= 1 - 2 - 3 = \boxed{-4}$

ex. Find: $f(2a)$

$f(2a) = (2a)^2 + 2(2a) - 3$
$\boxed{= 4a^2 + 4a - 3}$

2. Find: $f(5a)$

$f(5a) = (5a)^2 + 2(5a) - 3$
$\boxed{= 25a^2 + 10a - 3}$

Ex. Find: $f(a^2 - 2)$

$f(a^2-2) = (a^2-2)^2 + 2(a^2-2) - 3$
$= (a^2-2)(a^2-2) + 2a^2 - 4 - 3$
$a^4 - 4a^2 + 4 + 2a^2 - 4 - 3$
$= \boxed{a^4 + 2a^2 - 3}$

3. Find: $f(a^2 + 3)$

$f(a^2+3) = (a^2+3)^2 + 2(a^2+3) - 3$
$= (a^2+3)(a^2+3) + 2a^2 + 6 - 3$
$a^4 + 6a^2 + 9 + 2a^2 + 6 - 3$
$\boxed{= a^4 + 8a^2 + 12}$