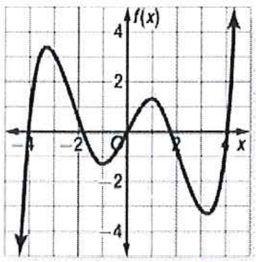
5.3 day 2 Investigating Graphs of Polynomial Functions

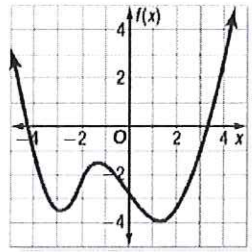
Key

Warm-Up:

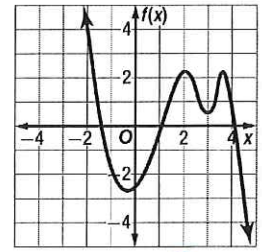
Describe the end behavior of each graph below, state the number of real zeros, and identify if odd or even degree:



$x \to +\infty,\ f(x) \to +\infty$
$x \to -\infty,\ f(x) \to -\infty$

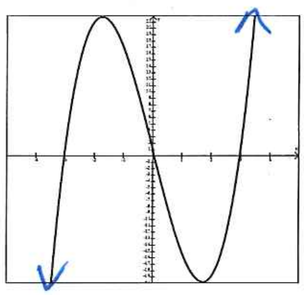


$x \to \pm\infty,\ f(x) \to +\infty$



$x \to +\infty,\ f(x) \to -\infty$
$x \to -\infty,\ f(x) \to +\infty$

We also need to be able to describe end behavior of a function by looking at its equation. The leading coefficient and the degree of the function affect end behavior.



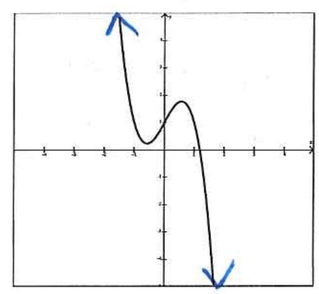
$f(x) = 2x^3 - 18x + 1$

Leading coefficient: 2

Degree of the function: 3 (odd)

Right end behavior: As $x \to \infty$, up

Left end behavior: As $x \to -\infty$, down



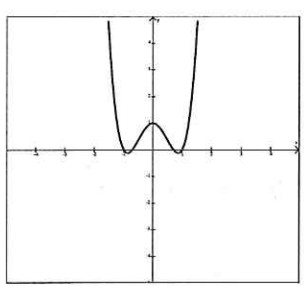
$f(x) = -2x^3 + 2x + 1$

Leading coefficient: -2

Degree of the function: 3 (odd)

Right end behavior: As $x \to \infty$, down

Left end behavior: As $x \to -\infty$, up



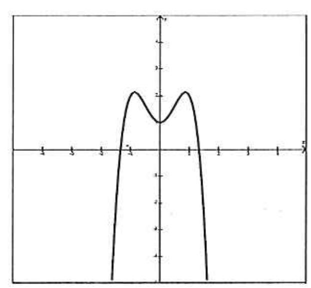
$f(x) = 2x^4 - 3x^2 + 1$

Leading coefficient: 2

Degree of the function: 4 (even)

Right end behavior: As $x \to \infty$, up

Left end behavior: As $x \to -\infty$, up



$f(x) = -2x^4 + 3x^2 + 1$

Leading coefficient: -2

Degree of the function: 4 (even)

Right end behavior: As $x \to \infty$, down

Left end behavior: As $x \to -\infty$, down

Rules for Determining End Behavior of Polynomial Functions:

If the leading coefficient is positive, the right end goes up.

If the leading coefficient is negative, the right end goes down

Even degree functions relate directly to a parabola and both the left & right ends go in the same direction.

Odd degree functions relate directly to a line and so the left end goes in the opposite direction as the right end.

Determine the ending behavior of each function based on its degree and leading coefficient.

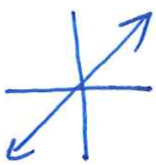
Ex. $p(x) = 3x^3 - x^2 + 2x - 5$

Degree: 3 (odd degree)

Leading coefficient: 3 (positive)

Ending behavior: $x \to +\infty, f(x) \to +\infty$

$x \to -\infty, f(x) \to -\infty$



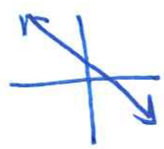
1. $p(x) = -x^5 + 4x^3$

Degree: 5 (odd degree)

leading coefficient: -1 (negative)

ending behavior: $x \to +\infty, f(x) \to -\infty$

$x \to -\infty, f(x) \to +\infty$

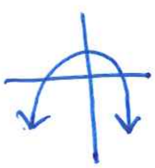


Ex: $p(x) = -7x^2 + 5x + 9$

Degree: 2 (even)

Leading coefficient: -7 (negative)

Ending behavior: $x \to \pm\infty, f(x) \to -\infty$



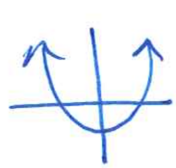
2. $f(x) = 3x^4 + 6x^3 - x^2 + 12$

Degree: 4 (even)

Leading coefficient: 3 (positive)

Ending behavior:

$x \to \pm\infty, f(x) \to +\infty$



Ex: $g(x) = 100 - 5x^3 + 10x^7$

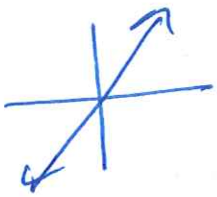
Standard form: $10x^7 - 5x^3 + 100$

Degree: 7 (odd)

Leading coefficient: 10 (positive)

Ending behavior: $x \to +\infty, f(x) \to +\infty$

$x \to -\infty, f(x) \to -\infty$



3. $p(x) = 4x^6 + 6x^4 + 8x^8 - 10x^2 + 20$

Standard form: $8x^8 + 4x^6 + 6x^4 - 10x^2 + 20$

Degree: 8 (even)

Leading coefficient: 8 (positive)

Ending behavior:

$x \to \pm\infty, f(x) \to +\infty$