5.6 Factor & Remainder Theorems

Objectives: Evaluate functions using synthetic substitution

Determine if given binomials are factors of a polynomial using synthetic division.

Remainder Theorem

THEOREM	EXAMPLE
If the polynomial function $P(x)$ is divided by $x - a$, then the remainder r is $P(a)$.	Divide $x^3 - 4x^2 + 5x + 1$ by $x - 3$. $\begin{array}{r\|rrrr} 3 & 1 & -4 & 5 & 1 \\ & & 3 & -3 & 6 \\ \hline & 1 & -1 & 2 & \boxed{7} \end{array}$ $P(3) = 7$

check: $P(3) = 3^3 - 4(3)^2 + 5(3) + 1$
$= 27 - 4(9) + 15 + 1$
$= 27 - 36 + 15 + 1$
$= 7$ ✓

Ex. Find $P(2)$ using the Remainder Theorem

$P(x) = 2x^4 + 5x^2 - x + 7$
$= 2x^4 + 0x^3 + 5x^2 - x + 7$

$$\begin{array}{r|rrrrr} 2 & 2 & 0 & 5 & -1 & 7 \\ + & \downarrow & 4 & 8 & 26 & 50 \\ \hline & 2 & 4 & 13 & 25 & \boxed{57} \end{array}$$

$P(2) = 57$

1. Find P (-2) if $P(x) = x^3 + 3x^2 + 4x - 5$

$$\begin{array}{r|rrrr} -2 & 1 & 3 & 4 & -5 \\ + & \downarrow & -2 & -2 & -4 \\ \hline & 1 & 1 & 2 & \boxed{-9} \end{array}$$

$P(-2) = -9$

Ex. Find $P(-1/3)$ using the Remainder Theorem for:

$P(x) = 6x^4 - 25x^3 - 3x + 5$

$$\begin{array}{r|rrrrr} -\frac{1}{3} & 6 & -25 & 0 & -3 & 5 \\ + & \downarrow & -2 & 9 & -3 & 2 \\ \hline & 6 & -27 & 9 & -6 & \boxed{7} \end{array}$$

$P(-1/3) = 7$

2. Find $P\left(-\frac{1}{2}\right)$ using the Remainder Theorem for:

$P(x) = 16x^3 - 8x^2 - 12x + 9$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 16 & -8 & -12 & 9 \\ + & \downarrow & -8 & 8 & 2 \\ \hline & 16 & -16 & -4 & \boxed{11} \end{array}$$

$P(-\frac{1}{2}) = 11$

Factor Theorem: If **(x - a)** is a factor of $p(x)$, then

1. $p(x)$ divided by **(x - a)** has a remainder of 0.
2. a is a zero of the function $p(x)$.

Ex) Use synthetic division to determine if $(x - 2)$ is a factor of $P(x) = x^3 - 7x^2 + 4x + 12$

If so, find the remaining factors and write the fully factored form of the function. (tip: each time we divide a polynomial by its factor, the quotient is one degree less than the original polynomial.) Check your work by entering the original equation in y1 and the factored form in y2. What do you notice?

$$\begin{array}{r|rrrr} 2 & 1 & -7 & 4 & 12 \\ + & \downarrow & 2 & -10 & -12 \\ \hline & 1 & -5 & -6 & \boxed{0} \end{array}$$

$(x-2)\underbrace{(x^2 - 5x - 6)}$

$P(x) = (x-2)(x-6)(x+1)$

remainder $= 0$ yes! $(x-2)$ is a factor

1) Is $(x-5)$ a factor of the polynomial, $x^3 - 7x^2 + 7x + 15$? If so, find the remaining factors and write the polynomial in its fully factored form.

$$\begin{array}{r|rrrr} 5 & 1 & -7 & 7 & 15 \\ + & \downarrow & 5 & -10 & -15 \\ \hline & 1 & -2 & -3 & \boxed{0} \end{array}$$

yes! remainder $= 0$ so $(x-5)$ is a factor!

$$(x-5)\underbrace{(x^2-2x-3)}$$

$$\boxed{(x-5)(x-3)(x+1)}$$

2) Is $(x+2)$ a factor of $P(x) = 3x^4 + 6x^3 - 5x + 10$? If so, write the fully factored form.

$$\begin{array}{r|rrrrr} -2 & 3 & 6 & 0 & -5 & 10 \\ + & \downarrow & -6 & 0 & 0 & 10 \\ \hline & 3 & 0 & 0 & -5 & \boxed{20} \end{array}$$

no! remainder $\neq 0$ so $(x+2)$ is not a factor

Ex) The volume of a rectangular box can be modeled by the function: $V(x) = x^3 + 11x^2 + 34x + 24$. One of the dimensions of the box can be represented by $(x+4)$. Write a function for the volume as a product of linear binomial factors.

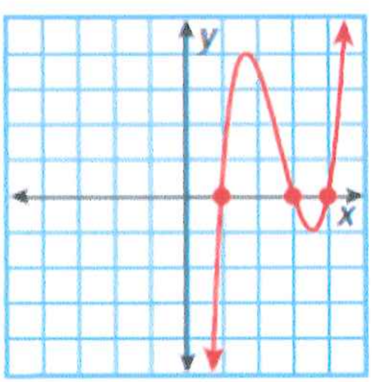
$$\begin{array}{r|rrrr} -4 & 1 & 11 & 34 & 24 \\ + & \downarrow & -4 & -28 & -24 \\ \hline & 1 & 7 & 6 & \boxed{0} \end{array}$$

$$(x-4)\underbrace{(x^2+7x+6)}$$

$$\boxed{V(x) = (x+4)(x+6)(x+1)}$$

Working backwards:

Ex) Write the simplest polynomial function for the graph given.



$x = 1$
$x = 3$
$x = 4$

$$(x-1)\underbrace{(x-3)(x-4)}$$

$$(x-1)(x^2-7x+12)$$

$$\begin{array}{rrrr} x^3 & -7x^2 & +12x & \\ & -x^2 & +7x & -12 \\ \hline \end{array}$$

$$\boxed{f(x) = x^3 - 8x^2 + 19x - 12}$$