

5-6 Practice Part 1
1e Remainder and Factor Theorems

Use synthetic substitution to find $f(-3)$ and $f(4)$ for each function.

1. $f(x) = x^2 + 2x + 3$

$$\begin{array}{r|rrr} -3 & 1 & 2 & 3 \\ & \downarrow & -3 & 3 \\ \hline & 1 & -1 & 6 \end{array} \quad f(-3) = 6$$

2. $f(x) = x^2 - 5x + 10$

$$\begin{array}{r|rrr} -3 & 1 & -5 & 10 \\ & \downarrow & -3 & 24 \\ \hline & 1 & -8 & 34 \end{array} \quad f(-3) = 34$$

$$\begin{array}{r|rrr} 4 & 1 & 2 & 3 \\ & \downarrow & 4 & 24 \\ \hline & 1 & 6 & 27 \end{array} \quad f(4) = 27$$

$$\begin{array}{r|rrr} 4 & 1 & -5 & 10 \\ & \downarrow & 4 & -4 \\ \hline & 1 & -1 & 6 \end{array} \quad f(4) = 6$$

3. $f(x) = x^3 - 2x^2 - 2x + 8$

$$\begin{array}{r|rrrr} -3 & 1 & -2 & -2 & 8 \\ & \downarrow & -3 & 15 & -39 \\ \hline & 1 & -5 & 13 & -31 \end{array} \quad f(-3) = -31$$

4. $f(x) = x^3 - x^2 + 4x - 4$

$$\begin{array}{r|rrrr} -3 & 1 & -1 & 4 & -4 \\ & \downarrow & -3 & 12 & -48 \\ \hline & 1 & -4 & 16 & -52 \end{array} \quad f(-3) = -52$$

$$\begin{array}{r|rrrr} 4 & 1 & -2 & -2 & 8 \\ & \downarrow & 4 & 8 & 24 \\ \hline & 1 & 2 & 6 & 32 \end{array} \quad f(4) = 32$$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & 4 & -4 \\ & \downarrow & 4 & 12 & 64 \\ \hline & 1 & 3 & 16 & 60 \end{array} \quad f(4) = 60$$

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

5. $x^3 + 3x^2 - 6x - 8; x - 2 = 0 \rightarrow x = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -6 & -8 \\ & \downarrow & 2 & 10 & 8 \\ \hline & 1 & 5 & 4 & 0 \end{array}$$

$x^2 + 5x + 4$
 $(x+5)(x+1)(x-2)$

6. $x^3 + 7x^2 + 7x - 15; x - 1$

$$\begin{array}{r|rrrr} 1 & 1 & 7 & 7 & -15 \\ & \downarrow & 1 & 8 & 15 \\ \hline & 1 & 8 & 15 & 0 \end{array}$$

$x^2 + 8x + 15$
 $(x+3)(x+5)(x-1)$

7. $3x^3 - 4x^2 - 17x + 6; x + 2$

$$\begin{array}{r|rrrr}
 -2 & 3 & -4 & -17 & 6 \\
 & \downarrow & & & \\
 & & -6 & 20 & -6 \\
 \hline
 & 3 & -10 & 3 & 0
 \end{array}$$

$3x^2 - 10x + 3$ $\begin{array}{r} 9 \\ -9, 1 \\ \hline 9 \end{array}$
 $(3x^2 - 9x)(-1x + 3)$
 $3x(x - 3) - 1(x - 3)$
 $\rightarrow (3x - 1)(x - 3)(x + 2)$

8. $4x^3 - 12x^2 - x + 3; x - 3$

$$\begin{array}{r|rrrr}
 3 & 4 & -12 & -1 & 3 \\
 & \downarrow & & & \\
 & & 12 & 0 & -3 \\
 \hline
 & 4 & 0 & -1 & 0
 \end{array}$$

$4x^2 - 1$ *Two terms difference of squares
 $(2x + 1)(2x - 1)(x - 3)$

9. **POPULATION** The projected population in thousands for a city over the next several years can be estimated by the function $P(x) = x^3 + 2x^2 - 8x + 520$, where x is the number of years since 2005. Use synthetic substitution to estimate the population for 2015.

$$\begin{array}{r|rrrr}
 10 & 1 & 2 & -8 & 520 \\
 & \downarrow & & & \\
 & & 10 & 120 & 1120 \\
 \hline
 & 1 & 12 & 112 & 1640
 \end{array}$$

$\rightarrow x = 10$

In 2015 the population will be 1640 thousand
1,640,000

10. **VOLUME** The volume of water in a rectangular swimming pool can be modeled by the polynomial $2x^3 - 9x^2 + 7x + 6$. If the depth of the pool is given by the polynomial $2x + 1$, what polynomials express the length and width of the pool?

$$\begin{array}{r|rrr}
 -\frac{1}{2} & 2 & -9 & 7 \\
 \hline
 & & &
 \end{array}$$



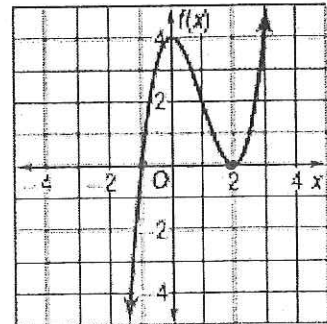
$$V = lwh$$

$$\frac{2x^3 - 9x^2 + 7x + 6}{2x + 1} = \frac{(l \cdot w)(2x + 1)}{2x + 1}$$

\downarrow
 $x = -\frac{1}{2}$

5.6 Practice PART 2

4. Write the equation in factored form from the graph.
Check your answer with Desmos, and then multiply your factors together to find the non-factored/standard form.

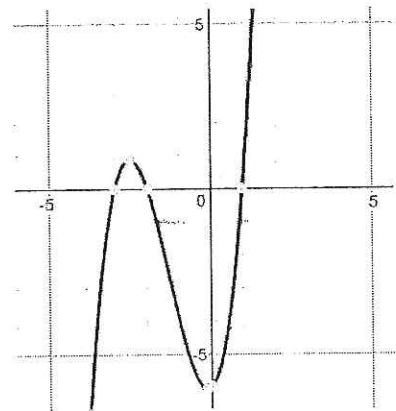


Check your work by graphing both the factored form & the non-factored form on the same grid in Desmos.

The two graphs should match.

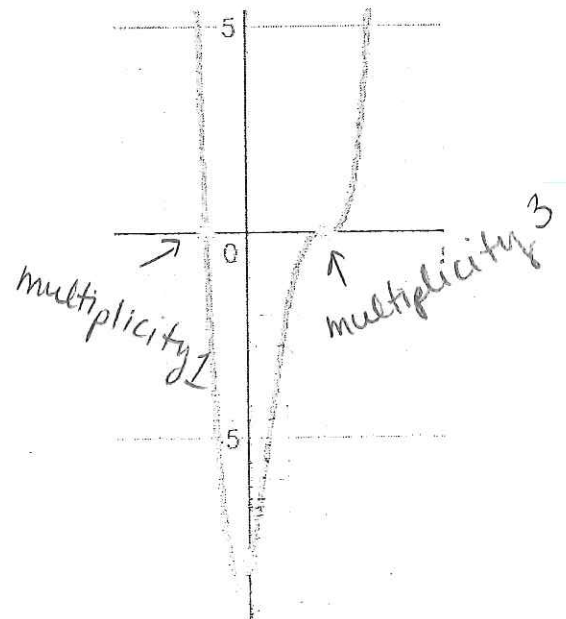
$$\begin{aligned}
 &x = -1 \qquad x = 2 \leftarrow \text{multiplicity of } 2 \\
 &(x+1)(x-2)^2 \\
 &(x+1)(x^2 - 4x + 4) \\
 &x^3 - 4x^2 + 4x + x^2 - 4x + 4 \\
 &\boxed{x^3 - 3x^2 + 4}
 \end{aligned}$$

5. Write the equation in factored form from the graph.
Multiply the factors to write the function in standard form.
Check your answer using Desmos to graph both forms on the same grid.



$$\begin{aligned}
 &x = -3 \qquad x = -2 \qquad x = 1 \\
 &(x+3)(x+2)(x-1) \\
 &(x^2 + 5x + 6)(x-1) \\
 &x^3 - x^2 + 5x^2 - 5x + 6x - 6 \\
 &\boxed{x^3 + 4x^2 + x - 6}
 \end{aligned}$$

6. Write the equation in factored form from the graph.
Multiply the factors to write the function in standard form.
Check your answer using Desmos to graph both forms on the same grid.



$$\begin{aligned}
 &x = -1 \qquad x = 2 \\
 &(x+1)(x-2)^3 \\
 &(x+1)(x-2)(x-2)(x-2) \\
 &(x^2 - x - 2)(x^2 - 4x + 4) \\
 &x^4 - 4x^3 + 4x^2 - x^3 + 4x^2 - 4x - 2x^2 + 8x - 8 \\
 &x^4 - 5x^3 + 6x^2 - 4x - 8
 \end{aligned}$$

