5.7 Notes Day 1: Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra: Every polynomial function of degree $n \geq 1$, has exactly n solutions. Solutions may be real complex, or repeated.

These statements are related:

If **k** is a zero of the polynomial function f(x), then:

1. **(x - k)** is a factor of the polynomial f(x)
2. **k** is a solution of the polynomial equation f(x) = 0
3. f(x) divided by **(x - k)** has a remainder of 0

Example: Suppose the zeros of f(x), a polynomial function, are: -3, 2, and 5

a) What is the minimum degree of the function? 3rd degree

b) What are the x-intercepts on the graph of f(x)? $x=-3$, $x=2$, $x=5$

c) What are the factors of the function? $(x+3)=0$ $(x-2)=0$ $(x-5)=0$

d) Write the equation of the function using the factors.

$(x+3)(x-2)(x-5)$

$(x+3)(x^2-7x+10)$

$= x^3-7x^2+10x+3x^2-21x+30$

$f(x)=x^3-4x^2-11x+30$

1) Suppose the zeros of p(x), a polynomial function, are: -1, 2, and 3

a) What is the minimum degree of the function? 3rd degree

b) What are the x-intercepts on the graph of f(x)? $x=-1$ $x=2$ $x=3$

c) What are the factors of the function? $(x+1)$ $(x-2)$ $(x-3)$

d) Write the equation of the function using the factors.

$f(x)=(x+1)(x^2-5x+6)$

x^3-5x^2+6x

$+x^2-5x+6$

$f(x)=x^3-4x^2+x+6$

Example: Solve: $x^4-3x^3+5x^2-27x-36=0$

a) Based on the degree, how many roots/solutions are there? 4 solutions

b) Graph the function and list the real roots. $x=-1$ & $x=4$

c) use the real roots and synthetic division to reduce the polynomial to a quadratic or lower

d) solve for remaining roots.

①

-1	1	-3	5	-27	-36
+	↓	-1	4	-9	36
	1	-4	9	-36	$\boxed{0}$

4	1	-4	9	-36
+	↓	4	0	36
	1	0	9	$\boxed{0}$

x^2+9

②

$x^2+9=0$

$\sqrt{x^2}=\sqrt{-9}$

$x=\pm i\sqrt{9}$

$=\pm 3i$

$x=-1$

$x=4$

$x=3i$

$x=-3i$

2) Solve: $x^4 + x^3 + 2x^2 + 4x - 8 = 0$

a) Based on the degree, how many roots/solutions are there? 4 Solutions

b) Graph the function and list the real roots. $x = -2$ & $x = 1$

c) Use the real roots and synthetic division to reduce the polynomial to a quadratic or lower

d) solve for remaining roots.

$$\begin{array}{r|rrrrr} -2 & 1 & 1 & 2 & 4 & -8 \\ + & \downarrow & -2 & 2 & -8 & 8 \\ \hline & 1 & -1 & 4 & -4 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 4 & -4 \\ + & \downarrow & 1 & 0 & 4 \\ \hline & 1 & 0 & 4 & \boxed{0} \end{array}$$

x^2+4

$x^2+4=0$

$x^2=-4$

$\sqrt{x^2} = \pm\sqrt{-4}$

$x = \pm 2i$

$x=-2$

$x=1$

$x=2i$

$x=-2i$

Multiplicity of a Root

Multiplicity of a Root: The multiplicity of a root is how many times the root is repeated.

Recognizing Multiplicity of a Root from a graph:

- Even multiplicity shows the function bouncing off the x axis
- Odd multiplicity > 1 shows the function 'flattening out' as it crosses the x axis

Find the roots of each function and state any multiplicity.

Ex. $f(x) = x^4 + 8x^3 + 17x^2 + 8x + 16$

a) Based on the degree, how many roots/solutions are there? 4 solutions

b) Graph the function and list the real roots, noting possible multiplicity of each. $x = -4$ w/ mult. of 2

c) use the real roots and synthetic division to reduce to quadratic or lower, then solve for remaining roots.

$$\begin{array}{r|rrrrr} -4 & 1 & 8 & 17 & 8 & 16 \\ + & \downarrow & -4 & -16 & -4 & -16 \\ \hline & 1 & 4 & 1 & 4 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} -4 & 1 & 4 & 1 & 4 \\ + & \downarrow & -4 & 0 & -4 \\ \hline & 1 & 0 & 1 & \boxed{0} \end{array}$$

x^2+1

$x^2+1=0$

$x^2=-1$

$\sqrt{x^2} = \pm\sqrt{-1}$

$x = \pm i$

Solutions

$x=-4$ w/ mult. of 2

$x = i$ $x = -i$

3) $f(x) = 3x^4 + 2x^3 - 67x^2 - 98x + 40$

a) Based on the degree, how many roots/solutions are there? 4 Solutions

b) Graph the function and list the real roots, noting possible multiplicity of each. $x=-4$ $x=-2$ $x=5$ $x=?$

c) use the real roots and synthetic division to reduce to quadratic or lower, then solve for remaining roots.

$$\begin{array}{r|rrrrr} -4 & 3 & 2 & -67 & -98 & 40 \\ + & \downarrow & -12 & 40 & 108 & -40 \\ \hline & 3 & -10 & -27 & 10 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} -2 & + \downarrow & -6 & 32 & -10 \\ \hline & 3 & -16 & 5 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrr} 5 & + \downarrow & 15 & -5 \\ \hline & 3 & -1 & \boxed{0} \end{array}$$

$3x - 1 = 0$

$+1 \quad +1$

$3x = 1$

$x = 1/3$

$x=-4$

$x=-2$

$x=5$

$x=1/3$