5.7 Notes Day 1: Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra: Every polynomial function of degree $n \geq 1$, has exactly n solutions. Solutions may be real complex, or repeated.

These statements are related:

If **k** is a zero of the polynomial function f(x), then:

1. **(x - k)** is a factor of the polynomial f(x)
2. **k** is a solution of the polynomial equation f(x) = 0
3. f(x) divided by **(x - k)** has a remainder of 0

Example: Suppose the zeros of f(x), a polynomial function, are: -3, 2, and 5

a) What is the minimum degree of the function?
b) What are the x-intercepts on the graph of f(x)?

c) What are the factors of the function?

d) Write the equation of the function using the factors.

1) Suppose the zeros of p(x), a polynomial function, are: -1, 2, and 3

a) What is the minimum degree of the function?
b) What are the x-intercepts on the graph of f(x)?

c) What are the factors of the function?

d) Write the equation of the function using the factors.

Example: Solve: $x^4 - 3x^3 + 5x^2 - 27x - 36 = 0$

a) Based on the degree, how many roots/solutions are there?

b) Graph the function and list the real roots.

c) use the real roots and synthetic division to reduce the polynomial to a quadratic or lower

d) solve for remaining roots.

2) Solve: $x^4 + x^3 + 2x^2 + 4x - 8 = 0$

a) Based on the degree, how many roots/solutions are there?

b) Graph the function and list the real roots.

c) Use the real roots and synthetic division to reduce the polynomial to a quadratic or lower

d) solve for remaining roots.

Multiplicity of a Root

Multiplicity of a Root: The multiplicity of a root is how many times the root is repeated.

Recognizing Multiplicity of a Root from a graph:

- Even multiplicity shows the function bouncing off the x axis
- Odd multiplicity > 1 shows the function 'flattening out' as it crosses the x axis

Find the roots of each function and state any multiplicity.

Ex. $f(x) = x^4 + 8x^3 + 17x^2 + 8x + 16$

a) Based on the degree, how many roots/solutions are there?

b) Graph the function and list the real roots, noting possible multiplicity of each.

c) use the real roots and synthetic division to reduce to quadratic or lower, then solve for remaining roots.

3) $f(x) = 3x^4 + 2x^3 - 67x^2 - 98x + 40$

a)Based on the degree, how many roots/solutions are there?

b) Graph the function and list the real roots, noting possible multiplicity of each.

c) use the real roots and synthetic division to reduce to quadratic or lower, then solve for remaining roots.