

# 5-7 Skills Practice PART 1

## Roots and Zeros

Solve each equation. State the number and type of roots.

1.  $5x + 12 = 0$

$5x = -12$

$x = \frac{-12}{5}$

1 real solution

2.  $x^2 - 4x + 40 = 0$  ← not factorable

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(40)}}{2(1)}$  Use quad formula  
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{4 \pm \sqrt{16 - 160}}{2}$

$x = \frac{4 \pm \sqrt{144}}{2} = \frac{4 \pm 12i}{2} = 2 \pm 6i$

2 imaginary solutions

3.  $x^5 + 4x^3 = 0$

$x^3(x^2 + 4) = 0$

$x^3 = 0$        $x^2 + 4 = 0$   
 $x^2 = -4$

$x = 0$        $x = \pm 2i$

mult of 3

3. 1 real (counts as 3)  
2 imaginary

4.  $x^4 - 625 = 0$

$(x^2)^2 - (25)^2 = 0$  \*use diff. of squares.

$(x^2 + 25)(x^2 - 25) = 0$   
 $a^2 - b^2 = (a+b)(a-b)$

not a difference       $(x+5)(x-5) = 0$   
 $x^2 + 25 = 0$        $x+5 = 0$        $x+5 = 0$

$x^2 = -25$        $x = -5$        $x = -5$

$x = \pm 5i$        $x = -5$        $x = -5$

2 imaginary      2 real

5.  $4x^2 - 4x - 1 = 0$

|       |      |
|-------|------|
| ac    | +b   |
| -4    | + -4 |
| 2, -2 | x    |
| none  |      |

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(-1)(4)}}{2(4)}$

$x = \frac{4 \pm \sqrt{16 + 16}}{8}$

$x = \frac{4 \pm \sqrt{32}}{8} = \frac{4 \pm 4\sqrt{2}}{8} = \frac{1 \pm \sqrt{2}}{2}$

2 real solutions

6.  $x^5 - 81x = 0$

$x(x^4 - 81) = 0$   
 $(x^2)^2 - (9)^2$  \*diff. of squares.

$x = 0$        $(x^2 + 9)(x^2 - 9)$   
 $(x^2 + 9)(x + 3)(x - 3)$

$x^2 + 9 = 0$        $x + 3 = 0$        $x - 3 = 0$   
 $x^2 = -9$        $x = -3$        $x = 3$   
 $x = \pm 3i$

3 real  
2 imaginary

Use your calculator to find all the rational real zeros of each function and then use synthetic division to reduce to a quadratic and solve.

7.  $h(x) = x^3 - 5x^2 + 5x + 3$

$x = 3 \leftarrow$  from calc.

$$\begin{array}{r|rrrr} 3 & 1 & -5 & 5 & 3 \\ & \downarrow & 3 & -6 & -3 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

$x^2 - 2x - 1$

$x = 3, 1 + \sqrt{2}, 1 - \sqrt{2}$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

8.  $g(x) = x^3 - 6x^2 + 13x - 10$

$x = 2 \leftarrow$  from calc.

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 13 & -10 \\ & \downarrow & 2 & -8 & 10 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$x^2 - 4x + 5$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i$$

$x = 2, 2 + i, 2 - i$

9.  $h(x) = x^3 + 4x^2 + x - 6$

$x = -3, -2, 1$

$\uparrow$   
all from calc  
degree is 3  
so need 3  
solutions

10.  $q(x) = x^3 + 3x^2 - 6x - 8$

$x = -4, 2, -1$

$\uparrow$   
all from calc.

# 5-7 Skills Practice PART 2

## Roots and Zeros

Use your calculator to find all the rational real zeros of each function and then use synthetic division to reduce to a quadratic and solve.

11.  $g(x) = x^4 - 3x^3 - 5x^2 + 3x + 4$

$X = -1 \quad X = 1 \quad X = 4$

  
 ↑  
 mult of 2

all from calc.

12.  $f(x) = x^4 - 21x^2 + 80$

$X = -4 \quad X = 4 \leftarrow$  from calc.

$$\begin{array}{r|rrrrr} -4 & 1 & 0 & -21 & 0 & 80 \\ & \downarrow & -4 & 16 & 20 & -80 \\ \hline 4 & 1 & -4 & -5 & 20 & 0 \\ & \downarrow & 4 & 0 & -20 & \\ \hline & 1 & 0 & -5 & 0 & \end{array}$$

$X^2 - 5 = 0$

$X^2 = 5$

$X = \pm\sqrt{5}$

$X = -4, 4, \sqrt{5}, -\sqrt{5}$

Write a polynomial function of least degree with integer coefficients that have the given zeros.

13.  $-3, -5, 1$

$$\begin{aligned} f(x) &= (x+3)(x+5)(x-1) \\ &= (x^2+5x+3x+15)(x-1) \\ &= (x^2+8x+15)(x-1) \\ &= x^3 - x^2 + 8x^2 - 8x + 15x - 15 \end{aligned}$$

$f(x) = x^3 + 7x^2 + 7x - 15$

integer

14.  $3i$

$X = \pm 3i$  \*to get rid of imaginary OR radical square both sides

$$\begin{aligned} X^2 &= (\pm 3i)^2 \\ X^2 &= 9i^2 \\ X^2 &= -9 \\ X^2 + 9 &= 0 \end{aligned}$$

$f(x) = X^2 + 9$

15.  $-5 \pm i$ 

$$X = -5 \pm i$$

$$(X+5)^2 = (\pm i)^2$$

$$(X+5)(X+5) = -1$$

$$X^2 + 5X + 5X + 25 = -1$$

$$X^2 + 10X + 25 = -1$$

$$X^2 + 10X + 26 = 0$$

$$f(x) = x^2 + 10x + 26$$

16.  $-1, \sqrt{3}, -\sqrt{3}$ 

$$X = -1$$

$$X = \pm\sqrt{3}$$

$$(X+1)$$

$$X^2 = (\pm\sqrt{3})^2$$

$$X^2 = 3$$

$$X^2 - 3 = 0$$

$$(X+1)(X^2 - 3)$$

$$X^3 - 3X + X^2 - 3$$

$$f(x) = x^3 + x^2 - 3x - 3$$

17.  $i, 5i$ 

$$X = \pm i \quad X = \pm 5i$$

$$X^2 = (\pm i)^2$$

$$X^2 = (\pm 5i)^2$$

$$X^2 = -1$$

$$X^2 = 25i^2$$

$$X^2 + 1 = 0$$

$$X^2 = -25$$

$$X^2 + 25 = 0$$

$$(X^2 + 1)(X^2 + 25)$$

$$X^4 + 25X^2 + X^2 + 25$$

$$f(x) = x^4 + 26x^2 + 25$$

18.  $-1, 1, i\sqrt{6}$ 

$$X = -1$$

$$X = 1$$

$$X = \pm i\sqrt{6}$$

$$(X+1) = 0$$

$$(X-1) = 0$$

$$X^2 = (\pm i\sqrt{6})^2$$

$$X^2 = i^2 \cdot 6$$

$$X^2 = -6$$

$$X^2 + 6 = 0$$

$$(X+1)(X-1)(X^2+6)$$

$$(X^2 - X + X - 1)(X^2 + 6)$$

$$(X^2 - 1)(X^2 + 6)$$

$$X^4 + 6X^2 - X^2 - 6$$

$$f(x) = x^4 + 5x^2 - 6$$