Key

6.1 Operations with Functions – Notes

Objectives: Adding, subtracting, multiplying and dividing functions
Finding compositions of functions
Understanding domain restrictions

Domain **(values of x) restrictions occur when: there is a variable in the denominator of a fraction or when there is a variable under a square root sign.

Example 1: $f(x) = x^2 + x - 6$ **and** $g(x) = x^2 - 4$

$(f+g)(x) = f(x) + g(x)$

$x^2 + x - 6 + x^2 - 4$

$= 2x^2 + x - 10$

D: ℝ

$(f-g)(x) = f(x) - g(x)$

$x^2 + x - 6 - (x^2 - 4)$

$= x^2 + x - 6 - x^2 + 4$

$= x - 2$

$(f \cdot g)(x) = f(x) \cdot g(x)$

$(x^2 - 4)(x^2 + x - 6)$

$x^4 + x^3 - 6x^2$

$+ \quad -4x^2 - 4x + 24$

$x^4 + x^3 - 10x^2 - 4x + 24$

D: ℝ

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2 + x - 6}{x^2 - 4}$

$= \frac{(x+3)(x-2)}{(x+2)(x-2)} = \frac{x+3}{x+2}$

D: $x \neq \pm 2$

Example 2: $f(x) = 3x^2$ **and** $g(x) = \frac{5}{x}$

$(f+g)(x) = f(x) + g(x)$

$= 3x^2 + \frac{5}{x}$

common denominators!

$\frac{3x^2}{1} \cdot \frac{x}{x} = \frac{3x^3}{x} + \frac{5}{x}$

$= \frac{3x^3 + 5}{x}$

D: $x \neq 0$

$(f-g)(x) \quad f(x) - g(x)$

D: $x \neq 0$

$= 3x^2 - \frac{5}{x}$

$= \frac{3x^3}{x} - \frac{5}{x} = \frac{3x^3 - 5}{x}$

$(f \cdot g)(x)$

$= f(x) \cdot g(x)$

$= \frac{3x^2}{1} \cdot \frac{5}{x}$

$= \frac{15x^2}{x} = 15x$

D: $x \neq 0$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x^2}{\frac{5}{x}} \rightarrow 3x^2 \div \frac{5}{x}$

$= 3x^2 \cdot \frac{x}{5} = \frac{3x^3}{5}$

D: $x \neq 0$

6.1 Day 1 Skills Practice

Exercises

1. $f(x) = x - 1; g(x) = 5x - 2$

$(f + g)(x) = x - 1 + 5x - 2$

$= 6x - 3$ D: $\mathbb{R}$

$(f - g)(x) =$

$x - 1 - (5x - 2) = -4x + 1$

$= x - 1 - 5x + 2$

$(f \cdot g)(x) = (x-1)(5x-2)$

$5x^2 \underbrace{- 2x - 5x} + 2$

D: $\mathbb{R}$

$5x^2 - 7x + 2$

$\left(\frac{f}{g}\right)(x) = \frac{x-1}{5x-2}$

D: $5x - 2 \neq 0$

$5x \neq 2$

$x \neq 2/5$

2. $f(x) = x^2 + x - 6; g(x) = x - 2$

$(f + g)(x) =$

$(x^2 + x - 6) + (x - 2) = x^2 + 2x - 8$

$x^2 + x - 6 + x - 2$

D: $\mathbb{R}$

$(f - g)(x) =$

$(x^2 + x - 6) - (x - 2) = x^2 - 4$

$x^2 \cancel{+x} - 6 \cancel{-x} + 2$

D: $\mathbb{R}$

$(f \cdot g)(x) = f(x) \cdot g(x)$

$(x - 2)(x^2 + x - 6)$

$x^3 + x^2 - 6x$

$-2x^2 - 2x + 12$

D: $\mathbb{R}$

$x^3 - x^2 - 8x + 12$

$\left(\frac{f}{g}\right)(x) = \frac{x^2 + x - 6}{x - 2} = \frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}}$

$= x + 3$

D: $x \neq 2$

3. $f(x) = x^2 - 1; g(x) = \frac{1}{x+1}$

$(f + g)(x) =$

$\frac{(x+1)}{(x+1)} \cdot \frac{(x^2 - 1)}{1} + \frac{1}{x+1}$

$= \frac{x^3 - x + x^2 - 1}{x+1} + \frac{1}{x+1} = \frac{x^3 + x^2 - x}{x+1}$

D: $x \neq -1$

$(f - g)(x) =$

$\frac{(x+1)}{(x+1)} \cdot \frac{x^2 - 1}{1} - \frac{1}{x+1}$

$= \frac{x^3 + x^2 - x - 2}{x+1}$ D: $x \neq -1$

$(f \cdot g)(x) =$

$x^2 - 1 \cdot \frac{1}{x+1}$

$= \frac{\cancel{(x+1)}(x-1)}{1} \cdot \frac{1}{\cancel{(x+1)}} = x - 1$

D: $x \neq -1$

$\left(\frac{f}{g}\right)(x) = \frac{x^2 - 1}{\frac{1}{x+1}} \Rightarrow x^2 - 1 \div \frac{1}{x+1}$

or $(x^2 - 1) \cdot \frac{(x+1)}{1}$

$= x^3 + x^2 - x - 1$

D: $x \neq -1$