Key

6.1 Day 2

Composition of Functions: $f \circ g(x)$ can be described by the equation $[f \circ g](x) = f[g(x)]$

In composition of functions, the output of the first (inside) function is used as the input to evaluate the second (outside) function. The range of the inside (first) function must be within the domain of the outside (second) function.

Example 4: **If** $f(x) = \{(1, 2), (3, 3), (2, 4), (4, 1)\}$ **and** $g(x) = \{(1, 3), (3, 4), (2, 2), (4, 1)\}$

Find: $f[g(1)] =$ $f(g(1)) = \boxed{3}$

$g(1) = 3$

$f(3) = 3$

$g[f(1)] =$ $f(1) = 2$ $g(f(1)) = \boxed{2}$

$g(2) = 2$

$f[g(2)] =$ $g(2) = 2$

$f(2) = 4$ $f(g(2)) = \boxed{4}$

$g[f(2)] =$ $f(2) = 4$ $\therefore g(f(2)) = \boxed{1}$

$g(4) = 1$

Find:: $f[g(3)] =$ $g(3) = 4$

$f(4) = 1$ $\therefore f(g(3)) = \boxed{1}$

$g[f(3)] =$ $f(3) = 3$ $\therefore g(f(3)) = \boxed{4}$

$g(3) = 4$

$f[g(4)] =$ $g(4) = 1$

$f(1) = 2$ $\therefore f(g(4)) = \boxed{2}$

$g[f(4)] =$ $f(4) = 1$ $\therefore g(f(4)) = \boxed{3}$

$g(1) = 3$

Example 5. If $f(x) = x + 4$, and $g(x) = x^2 - 1$, **find each value.**

$f[g(1)] =$

$g(1) = 1^2 - 1 = 0$

$f(0) = 0 + 4 = \boxed{4}$

$g[f(1)] =$

$f(1) = 5$

$g(5) = \boxed{24}$

$f[g(2)] =$

$g(2) = 3$

$f(3) = \boxed{7}$

$g[f(2)] =$ $f(2) = 6$

$g(6) = \boxed{35}$

Example 6. If $f(x) = 5x + 4$; $g(x) = 3 - x$, **find:**

$[f \circ g](x) =$ $f(g(x))$

$= f(3-x)$

$= 5(3-x) + 4$ $= \boxed{19-5x}$

$= 15 - 5x + 4$

$[g \circ f](x) =$ $g(f(x))$

$= g(5x+4)$

$= 3 - (5x+4)$ $= \boxed{-5x-1}$

$= 3 - 5x - 4$

Example 7. If $g(x) = 3x - 4$ **and** $h(x) = x^2 - 1$, **find:**

$[g \circ h](x) =$ $g(h(x))$

$= g(x^2-1) = 3(x^2-1) - 4$

$= 3x^2 - 3 - 4$

$= \boxed{3x^2 - 7}$

$[h \circ g](x) =$ $h(g(x))$

$= h(3x-4)$

$= (3x-4)^2 - 1$

$= (3x-4)(3x-4) - 1$

$9x^2 - 24x + 16 - 1$

$\boxed{= 9x^2 - 24x + 15}$

6.1 Day 2 Skills Practice

Find $[f \circ g](x)$ and $[g \circ f](x)$

4. $f(x) = -3x$; $g(x) = -x + 8$

$f(g(x)) = f(-x+8)$
$= -3(-x+8)$
$= \boxed{3x - 24}$

$g(f(x)) = g(-3x)$
$= -(-3x) + 8$
$= \boxed{3x + 8}$

5. $f(x) = x^2 - 1$; $g(x) = -4x^2$

$f(g(x)) = f(-4x^2)$
$= (-4x^2)^2 - 1$
$\boxed{= 16x^4 - 1}$

$g(f(x)) = g(x^2-1)$
$= -4(x^2-1)^2$
$= -4[(x^2-1)(x^2-1)]$
$= -4(x^4 - 2x^2 + 1)$
$\boxed{= -4x^4 + 8x^2 - 4}$

6. $f(x) = x^2 + 2x$; $g(x) = x - 9$

$f(g(x)) = f(x-9)$
$= (x-9)^2 + 2(x-9)$
$= x^2 - 18x + 81 + 2x - 18$
$\boxed{= x^2 - 16x + 63}$

$g(f(x)) = g(x^2+2x)$
$= (x^2+2x) - 9$
$= \boxed{x^2 + 2x - 9}$

7. $f(x) = 8x^2 + 3x$; $g(x) = 2x^2$

$f(g(x)) = f(2x^2)$
$= 8(2x^2)^2 + 3(2x^2)$
$= 8(4x^4) + 6x^2$
$= \boxed{32x^4 + 6x^2}$

$g(f(x)) = g(8x^2 + 3x)$
$= 2(8x^2 + 3x)$
$= \boxed{16x^2 + 6x}$

If $f(x) = 3x$, $g(x) = x + 4$, and $h(x) = x^2 - 1$, find each value.

8. $f[g(1)]$

$g(1) = 5$
$f(5) = \boxed{15}$

9. $g[h(0)]$

$h(0) = -1$
$g(-1) = \boxed{3}$

10. $g[f(-1)]$

$f(-1) = -3$
$g(-3) = \boxed{11}$

11. $h[f(5)]$

$f(5) = 15$
$h(15) = \boxed{224}$

12. $g[h(-3)]$

$h(-3) = 8$
$g(8) = \boxed{12}$

13. $h[f(10)]$

$f(10) = 30$
$h(30) = \boxed{899}$

If $f = \{(0,0), (4,-2)\}$ *and* $g = \{(0,4), (-2,0), (5,0)\}$, find:

14. $f[g(5)]$

$g(5) = 0$
$f(0) = \boxed{0}$

15. $g[f(4)]$

$f(4) = -2$
$g(-2) = \boxed{0}$