6.4 notes - n^{th} roots

Objectives: Simplifying radicals

Key

Squaring a number and taking the square root are inverse operations - they 'undo' one another. Similarly:

The inverse of raising a number to the n^{th} power is finding the n^{th} root.

Examples:

Powers	Factors	Roots
$x^3 = 64$	$4 * 4 * 4 = 64$	$\sqrt[3]{64} = 4$
$x^4 = 625$	$5 \cdot 5 \cdot 5 \cdot 5 = 625$	$\sqrt[4]{625} = 5$
$x^5 = 32$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$	$\sqrt[5]{32} = 2$

!!!!! **Remember:** Unless we insert the radical sign when solving an equation, we always default to the principal (positive) root.

Check it out:

1. $\sqrt[4]{16} = \boxed{2}$

$2 \cdot 2 \cdot 2 \cdot 2 = 16$

2. $\sqrt[3]{27} = \boxed{3}$

$3 \cdot 3 \cdot 3 = 27$

3. $\sqrt[3]{-27} = \boxed{-3}$

$-3 \cdot -3 \cdot -3 = -27$

4. $\sqrt{16x^4} = \boxed{4x^2}$

$\sqrt{16} = 4$

$\sqrt{x^4} = x^2$

5. $\sqrt[5]{-32} = \boxed{-2}$

$\underbrace{-2 \cdot -2 \cdot -2 \cdot -2 \cdot -2}$

$= -32$

6. $\sqrt[4]{-81}$ Does not exist!

$(-3)^4 = 81$

$(3)^4 = 81$

7. $\sqrt[3]{-8x^{12}} = \boxed{-2x^4}$

$\sqrt[3]{-8} = -2$

$\sqrt[3]{x^{12}} = x^4$

8. $\sqrt{-16}$ - DNE even index root.

9. $\sqrt[4]{81x^4y^8} = \boxed{3xy^2}$

$\sqrt[4]{81} = 3$

$\sqrt[4]{x^4} = x$

$\sqrt[4]{y^8} = y^2$

10. $\sqrt[5]{243x^{20}y^{25}} = \boxed{3x^4y^5}$

$\sqrt[5]{243} = 3$

$\sqrt[5]{x^{20}} = x^4$

$\sqrt[5]{y^{25}} = y^5$

11. $\sqrt[2]{(x^2-6)^8}$

$\boxed{= (x^2-6)^4}$

index of the root $\rightarrow$

Summary: $\sqrt[n]{a}$ $\leftarrow$ radicand

If the index of the root **(n) is even** and the radicand **(a) is positive,** the principal root is always positive.

If the index of the root **(n) is even,** and the radicand **(a) is negative** there is no real solution

If the index of the root **(n) is odd,** the principal root can be **positive or negative,** based on whether the radicand is positive or negative.

Sometimes n^{th} roots are preceded by signs.

If something other than the principal root is desired, the radical sign will be preceded by signs.

Ex. Simplify.

a) $\sqrt{25} =$ 5 **b)** $-\sqrt{25} =$ -5 **c)** $\pm\sqrt{25} =$ ± 5

Practice.

13. $\pm\sqrt{36x^{12}} =$

$\sqrt{36} = 6$

$\sqrt{x^{12}} = x^6$

$\boxed{\pm 6x^6}$

14. $-\sqrt{(y+7)^{16}}$

$\boxed{= -(y+7)^8}$

15. $\sqrt[6]{d^{24}x^{36}}$ $= \boxed{d^4 x^6}$

$\sqrt[6]{d^{24}} = d^4$

$\sqrt[6]{x^{36}} = x^6$

16. $-\sqrt[4]{625y^8} =$

$\sqrt[4]{625} = 5$

$\sqrt[4]{y^8} = y^2$

$\boxed{-5y^2}$

17. $\sqrt[5]{-32x^5y^{10}}$

$\sqrt[5]{-32} = -2$

$\sqrt[5]{x^5} = x$

$\sqrt[5]{y^{10}} = y^2$

$= \boxed{-2xy^2}$

18. $\pm\sqrt{\frac{9m^4}{25}}$

$\sqrt{9} = 3$

$\sqrt{m^4} = m^2$

$\sqrt{25} = 5$

$\boxed{= \pm\frac{3m^2}{5}}$

19. $\sqrt[3]{27x^6}$

$\sqrt[3]{27} = 3$

$\sqrt[3]{x^6} = x^2$

$= \boxed{3x^2}$

20. $\sqrt[4]{16(x-3)^{12}}$

$\sqrt[4]{16} = 2$

$\sqrt[4]{(x-3)^{12}} = (x-3)^3$

$\boxed{= 2(x-3)^3}$

21. $\sqrt{x^2 + 10x + 25}$

$= \sqrt{(x+5)(x+5)}$

$= \sqrt{(x+5)^2}$

$= \boxed{x+5}$