

6.6 Rational Exponents & Radicals - Day 1 Notes

Objective: Writing expressions with rational exponents and in radical form.
Simplifying expressions in exponential or radical form

Rational Exponents & Radicals

$$\sqrt[n]{b} = b^{\frac{1}{n}}$$

Index of the root is ALWAYS on the **bottom** of rational exponents!

Examples. $\sqrt[3]{27} = 27^{1/3}$

$$(16)^{\frac{1}{2}} = \sqrt{16}$$

Write each example in radical or exponential form.

1. $\sqrt[4]{16} =$

$$16^{1/4}$$

$$= 2$$

2. $\sqrt[3]{-216} =$

$$(-216)^{1/3}$$

$$= -6$$

3. $\sqrt{121} =$

$$(121)^{1/2}$$

$$= 11$$

4. $256^{\frac{1}{2}} =$

$$\sqrt{256}$$

$$= 16$$

5. $32^{\frac{1}{5}} =$

$$\sqrt[5]{32}$$

$$= 2$$

6. $(-64)^{\frac{1}{3}} =$

$$\sqrt[3]{-64}$$

$$= -4$$

Definition: $b^{\frac{m}{n}} = (\sqrt[n]{b})^m$ or $b^{\frac{m}{n}} = \sqrt[n]{b^m}$

Examples. $27^{\frac{2}{3}} =$

$$\left(\sqrt[3]{27}\right)^2 = 3^2 = 9$$

or

$$27^{\frac{2}{3}} =$$

$$\sqrt[3]{27^2} = \sqrt[3]{729} = 9$$

Write each example in radical or exponential form.

7. $16^{\frac{3}{2}} =$

$$\left(\sqrt{16}\right)^3$$

$$= 4^3 = 64$$

8. $(-27)^{\frac{4}{3}} =$

$$\left(\sqrt[3]{-27}\right)^4 = (-3)^4 = 81$$

9. $\sqrt[4]{15^3} =$

$$15^{3/4}$$

10. $(\sqrt[5]{-32})^3 =$

$$(-32)^{3/5}$$

11. $2^{\frac{5}{6}} =$

$$\left(\sqrt[6]{2}\right)^5$$

or $\sqrt[6]{2^5}$

12. $(\sqrt[3]{8x^9})^5 =$

$$(8x^9)^{5/3}$$

Negative Rational Exponents

$$16^{-\frac{1}{2}} = \frac{1}{16^{\frac{1}{2}}}$$

Rewrite each expression using positive rational exponents. Simplify if possible.

13. $16^{\frac{1}{4}} =$

$$\frac{1}{16^{-1/4}} = \frac{1}{4\sqrt[4]{16}} = \boxed{\frac{1}{2}}$$

14. $64^{\frac{1}{3}} =$

$$\frac{1}{64^{-1/3}} = \frac{1}{\sqrt[3]{64}} = \boxed{\frac{1}{4}}$$

15. $(-32)^{\frac{1}{5}} =$

$$\frac{1}{(-32)^{-1/5}} = \frac{1}{\sqrt[5]{-32}} = \boxed{-\frac{1}{2}}$$

All the Rules for Integer Exponents Also Apply to Rational Exponents

Simplify each expression. Translate between rational and radical form.

16. $27^{\frac{1}{3}} \cdot 27^{\frac{5}{3}}$

$$= 27^{1/3 + 5/3} = 27^{6/3} = 27^2 = \boxed{729}$$

17. $\left(\frac{4}{9}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{4}{9}}\right)^3$

$$= \left(\frac{2}{3}\right)^3 = \boxed{\frac{8}{27}}$$

18. $c^{\frac{12}{5}} \cdot c^{\frac{3}{5}}$

$$= c^{12/5 + 3/5} = c^{15/5} = \boxed{c^3}$$

19. $x^{\frac{6}{11}} \cdot x^{\frac{5}{11}}$

$$= x^{6/11 + 5/11} = x^{11/11} = \boxed{x}$$

20. $\left(\frac{125}{216}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{125}{216}}\right)^2$

$$= \left(\frac{5}{6}\right)^2 = \boxed{\frac{25}{36}}$$

21. $\frac{q^{\frac{3}{2}}}{q^{\frac{5}{2}}}$

$$= q^{3/2 - 5/2} = q^{-2/2} = q^{-1} = \frac{1}{q} = \boxed{\frac{1}{q}}$$

22. $\frac{x^{\frac{3}{4}}}{x^4} = x$

$$= x^{3/4 - 4} = x^{-13/4} = \frac{1}{x^{13/4}} = \boxed{\frac{1}{x^3 \sqrt[4]{x}}}$$

23. $(5^{\frac{1}{3}})^6 = 5^{\left(\frac{1}{3}\right)^6}$

$$= 5^2 = 25 = \boxed{25}$$

24. $(3^{\frac{2}{5}})^{10}$

$$= 3^{2/5 \cdot 10} = 3^4 = \boxed{81}$$