6.6 Rational Exponents – Day 2 Notes

Objective: Simplifying different nth root radical and rational expressions.

Recall that when in radical form, we were only able to add, subtract, multiply & divide radical expressions with matching indexed roots.

Ex. $\frac{\sqrt[3]{16}}{\sqrt[3]{2}} =$ $\sqrt[3]{\frac{16}{2}} = \sqrt[3]{8}$ $\boxed{= 2}$

ex. $\sqrt{2} + \sqrt{32} - \sqrt{8}$

$\sqrt{2} + \sqrt{16 \cdot 2} - \sqrt{4 \cdot 2}$

$\sqrt{2} + 4\sqrt{2} - 2\sqrt{2}$ $= \boxed{4\sqrt{2}}$

Translating radical expressions into <u>rational exponents</u> allows us to perform operations with radicals <u>with different index roots.</u>.

General Rule for simplifying radicals: final answer should be in same form as the original problem.

Write as one term. Simplify if possible.

Ex. $\sqrt[5]{p} * \sqrt{p} =$

$p^{1/5} \cdot p^{1/2} = p^{1/5 + 1/2}$

$= p^{2/10 + 5/10} = p^{7/10}$ or $\boxed{\sqrt[10]{p^7}}$

1. $\sqrt[6]{16} * \sqrt[3]{16} =$ $16^{1/6} \cdot 16^{1/3} = 16^{1/6 + 1/3}$

$16^{1/6 + 2/6} = 16^{3/6}$ or $16^{1/2}$

$\boxed{\sqrt{16} = 4}$

Ex. $\frac{x^{\frac{2}{3}}}{x^{\frac{1}{4}}}$ $x^{2/3 - 1/4}$

$= x^{8/12 - 3/12} = \boxed{x^{5/12}}$

2. $\frac{64^{\frac{3}{2}}}{64^{\frac{4}{3}}}$ $64^{3/2 - 4/3}$

$= 64^{9/6 - 8/6} = \boxed{64^{1/6}}$

3. $(\sqrt[5]{x})^2 * (\sqrt[10]{x})^3$ $x^{2/5 + 3/10}$

$= x^{4/10 + 3/10}$

$= x^{7/10}$ or $\boxed{\sqrt[10]{x^7}}$

4. $\frac{p}{p^{\frac{1}{3}}}$ $p^{1 - 1/3} = p^{3/3 - 1/3}$

$= \boxed{p^{2/3}}$

Bases must match when translating to rational exponents to multiply or divide different indexed root radicals.

Ex. $\sqrt[3]{9} * \sqrt{27}$

$\sqrt[3]{3^2} \cdot \sqrt{3^3} = 3^{2/3} \cdot 3^{3/2}$

$= 3^{2/3+3/2} = 3^{4/6+9/6}$ $\boxed{3^{13/6}}$

Simplify! $3^{\frac{12}{6}} \cdot 3^{1/6}$

or $\boxed{9\sqrt[6]{3}}$

5. $\sqrt[4]{8} * \sqrt{2}$

$\sqrt[4]{2^3} \cdot \sqrt{2} = 2^{3/4} \cdot 2^{1/2}$

$= 2^{3/4+1/2} = 2^{3/4+2/4}$

$= 2^{5/4}$ or $\boxed{2\sqrt[4]{2}}$

Ex. $\frac{\sqrt[4]{27}}{\sqrt{3}}$ $= \frac{\sqrt[4]{3^3}}{\sqrt{3}} = \frac{3^{3/4}}{3^{1/2}}$

$= 3^{3/4-1/2}$

$= 3^{3/4-2/4} = 3^{1/4}$

$\boxed{= \sqrt[4]{3}}$

6. $\frac{\sqrt[4]{32}}{\sqrt[3]{2}}$ $\frac{\sqrt[4]{2^5}}{\sqrt[3]{2}} = \frac{2^{5/4}}{2^{1/3}}$

$= 2^{5/4-1/3}$

$= 2^{15/12-4/12}$

$\boxed{= 2^{11/12} \text{ or } (\sqrt[12]{2})^{11}}$

7. $\frac{\sqrt[6]{16}}{\sqrt[3]{2}}$ $\frac{\sqrt[6]{2^4}}{\sqrt[3]{2}} = \frac{2^{4/6}}{2^{1/3}}$

$= 2^{2/3-1/3}$

$= 2^{1/3}$ or $\boxed{\sqrt[3]{2}}$

Special cases with negative exponents and rationalizing denominators.

Ex. $b^{-\frac{5}{6}}$ $\frac{1}{b^{5/6}}$

$\frac{1}{\sqrt[6]{b^5}} \cdot \frac{\sqrt[6]{b}}{\sqrt[6]{b}} = \boxed{\frac{\sqrt[6]{b}}{b} \text{ or } \frac{b^{1/6}}{b}}$

9. $r^{-\frac{4}{5}}$ $\frac{1}{r^{4/5}}$

$= \frac{1}{\sqrt[5]{r^4}} \cdot \frac{\sqrt[5]{r}}{\sqrt[5]{r}} = \boxed{\frac{\sqrt[5]{r}}{r} \text{ or } \frac{r^{1/5}}{r}}$

10. $b^{-\frac{1}{4}}$ $= \frac{1}{b^{1/4}}$

$\frac{1}{\sqrt[4]{b}} \cdot \frac{\sqrt[4]{b^3}}{\sqrt[4]{b^3}} = \boxed{\frac{\sqrt[4]{b^3}}{b}}$

11. $x^{-\frac{2}{5}}$ $= \frac{1}{x^{2/5}}$

$\frac{1}{\sqrt[5]{x^2}} \cdot \frac{\sqrt[5]{x^3}}{\sqrt[5]{x^3}} = \boxed{\frac{\sqrt[5]{x^3}}{x}}$

or $\frac{x^{3/5}}{x}$