

7.2 Day 2 Notes Solving Exponential Equations & Inequalities

Key

Objectives: Solving exponential equations & inequalities using same base formula

An exponential equation is an equation containing one or more expressions that have

variables as an exponent.

To solve exponential equations: write with same bases if possible

1) Writing them so that the bases are all the same.

If $b^x = b^y$, then $x = y$

Solve for x:

Ex. $3^x = 3^2$

$x = 2$

a) $3^{2x-1} = 3^{x+2}$

$2x - 1 = x + 2$
 $\underline{-x}$

$x - 1 = 2$
 $\underline{+1}$

$x = 3$

Ex. $8^x = 2^{x+6}$

$(2^3)^x = 2^{x+6}$
 $2^{3x} = 2^{x+6}$
 $\underline{-x}$

$3x = x + 6$
 $\underline{-x}$

$2x = 6$
 $x = 3$

b) $25^{x-2} = 125$

$(5^2)^{x-2} = 5^3$
 $5^{2x-4} = 5^3$

$2x - 4 = 3$
 $2x = 7$

$x = 7/2$

Ex. $3^{2x-1} = \frac{1}{9}$

$3^{2x-1} = 3^{-2}$
 $2x - 1 = -2$
 $\underline{+1}$
 $2x = -1$
 $\underline{2}$

$x = -\frac{1}{2}$

c) $(\frac{1}{36})^{2x} = 6^5$

$(6^{-2})^{2x} = 6^5$
 $6^{-4x} = 6^5$

$-4x = 5$
 $\underline{-4}$
 $x = -\frac{5}{4}$

d) $36^{2x+4} = 216^{x+5}$

$(6^2)^{2x+4} = (6^3)^{x+5}$
 $6^{4x+8} = 6^{3x+15}$
 $4x + 8 = 3x + 15$
 $\underline{-3x}$
 $x + 8 = 15$
 $\underline{-8}$
 $x = 7$

e) $(\frac{1}{64})^{x-2} = 16^{3x+1}$

$(4^{-3})^{x-2} = (4^2)^{3x+1}$
 $4^{-3x+6} = 4^{6x+2}$
 $-3x + 6 = 6x + 2$
 $\underline{-6x}$
 $-9x + 6 = 2$
 $\underline{-6}$
 $-9x = -4$
 $\underline{-9}$

$x = 4/9$

Solving Exponential Inequalities

If $b > 1$, $b^x > b^y$ if and only if $x > y$. Example: $2^x > 2^6$ if and only if $x > 6$.

Solve:

Ex. $16^{2x-3} > 8$

$$\begin{aligned}
 (2^4)^{2x-3} &> 2^3 \\
 2^{8x-12} &> 2^3 \\
 8x-12 &> 3 \\
 8x-12 &+12 > 3+12 \\
 8x &> 15 \\
 x &> \frac{15}{8}
 \end{aligned}$$

1. $625 \geq 5^{a+8}$

$$\begin{aligned}
 5^4 &\geq 5^{a+8} \\
 4 &\geq a+8 \\
 -8 & \\
 -4 &\geq a \\
 a &\leq -4
 \end{aligned}$$

Ex. $\left(\frac{1}{9}\right)^{3t+5} \geq \left(\frac{1}{243}\right)^{t-6}$

$$\begin{aligned}
 (3^{-2})^{3t+5} &\geq (3^{-5})^{t-6} \\
 3^{-6t-10} &\geq 3^{-5t+30}
 \end{aligned}$$

$$\begin{aligned}
 -6t-10 &\geq -5t+30 \\
 +5t & \quad +5t
 \end{aligned}$$

$$\begin{aligned}
 -t-10 &\geq 30 \\
 +10 & \quad +10
 \end{aligned}$$

$$\begin{aligned}
 -t &\geq 40 \\
 -1 & \quad -1
 \end{aligned}$$

$$t \leq -40$$

2. $\left(\frac{1}{27}\right)^{2d-2} < 81^{d+4}$

$$\begin{aligned}
 (3^{-3})^{2d-2} &< (3^4)^{d+4} \\
 3^{-6d+6} &< 3^{4d+16}
 \end{aligned}$$

$$\begin{aligned}
 -6d+6 &< 4d+16 \\
 +6d & \quad +6d
 \end{aligned}$$

$$\begin{aligned}
 6 &< 10d+16 \\
 -16 & \quad -16
 \end{aligned}$$

$$\begin{aligned}
 -10 &< 10d \\
 10 & \quad 10
 \end{aligned}$$

$$d > -1$$