

7.4 Notes - Logarithmic Equations & Inequalities

* $x =$ the argument

argument = answer to exponential

Use what you know about logarithms to determine what value of x will make the following equations true.

& Always Positive

1. $\log_6(x) = 2$

$6^2 = x$
 $x = 36$

2. $\log_3(x) = 4$

$3^4 = x$
 $x = 81$

3. $\log_{\frac{1}{5}}(x) = -3$

$(\frac{1}{5})^{-3} = x$
 $x = 125$

4. $\log_{36}(x) = \frac{1}{2}$

$36^{\frac{1}{2}} = x$
 $x = 6$

5. $\log_{0.16}(x) = 0$

$0.16^0 = x$
 $x = 1$

6. $\log_2(x) = -4$

$2^{-4} = x$
 $x = \frac{1}{16}$

Extension.

7. Solve: $\log_6(2x - 4) = 2$

$6^2 = 2x - 4$
 $36 = 2x - 4$
 $+4$ $+4$
 $40 = 2x$
 $x = 20$

8. Solve: $\log_3(5x + 7) = 3$

$3^3 = 5x + 7$
 $27 = 5x + 7$
 -7 -7
 $20 = 5x$
 $\frac{20}{5} = \frac{5x}{5}$
 $x = 4$

Property of equality: If $\log_b(x) = \log_b(y)$, then $x = y$.

ex. Solve: $\log_2(x^2 - 4) = \log_2(3x)$

check:

factor!
 $x^2 - 4 = 3x$
 $x^2 - 3x - 4 = 0$
 $(x - 4)(x + 1) = 0$
 $x - 4 = 0$ $x + 1 = 0$
 $x = 4$ $x = -1$

Always check solutions. They can't make the argument negative!

$\log_2(3(-1))$
 $\log_2(-3)$
undefined!

9. solve: $\log_3(x^2 + 6) = \log_3(-5x)$

check:

$x^2 + 6 = -5x$
 $x^2 + 5x + 6 = 0$
 $(x + 3)(x + 2) = 0$
 $x + 3 = 0$ $x + 2 = 0$
 $x = -3$ $x = -2$

$\log_3((-3)^2 + 6) = \log_3(-5(-3))$
 $\log_3 15 = \log_3 15$ ✓
 $\log_3[(-2)^2 + 6] = \log_3[-5(-2)]$
 $\log_3 10 = \log_3 10$ ✓

Properties of Inequality: If $b > 1$

$\log_b(x) > y$, then $x > b^y$

$\log_b(x) < y$, then $x < b^y$

Example:

Solve. $\log_3(x) > 2$

$x > 3^2$
 $x > 9$

8. Solve. $\log_4(x) > 3$

$x > 4^3$
 $x > 64$

Example:

Solve. $\log_2(x) \leq 4$

$x \leq 2^4$
 $x \leq 16$
 $0 < x \leq 16$
 or
 $(0, 16]$

* Don't forget:
 Argument must be > 0 .

9. Solve. $\log_5(x) \leq 2$

$x \leq 5^2$
 $x \leq 25$
 and
 $x > 0$
 $(0, 25]$

Extension.

10. Solve: $\log_2(x + 6) \leq 4$

$x + 6 \leq 2^4$
 $x + 6 \leq 16$
 $x \leq 10$

Argument > 0

$x + 6 > 0$
 $x > -6$

$(-6, 10]$

11. Solve: $\log_4(3x - 8) \leq 3$

$3x - 8 \leq 4^3$
 $3x - 8 \leq 64$
 $3x \leq 72$
 $x \leq 24$

Argument > 0

$3x - 8 > 0$
 $3x > 8$
 $x > \frac{8}{3}$

$(\frac{8}{3}, 24]$