No $a \pm \sqrt{b}$?

original 20, 21 (18, 19) new

Name: Key **Period:** ______

5.5 – 5.7 TEST Review

Factor completely. Write in complete factored form

1. $5x^4 + 40x$

$5x(x^3+8)$

$5x(x+2)(x^2-2x+4)$

2. $3x^2 + 13x - 10$

-30, 13; $15, -2$

$(3x^2+15x)+(-2x-10)$

$3x(x+5)-2(x+5)$

$(x+5)(3x-2)$

3. $2x^3 - 12x^2 - 32x + 192$

$2(x^3-6x^2)+(-16x+96)$

$2x^2(x-6)+-16(x-6)$

$2(x^2-8)(x-6)$

4. $x^4 - 29x^2 + 100$

$(x^2)^2 - 29(x^2) + 100$

$(x^2-25)(x^2-4)$

100, 29; 25, 4

$(x+5)(x-5)(x+2)(x-2)$

108; 2, 54; 2, 17

Factor & solve each equation.

5. $6x^5 + 48x^2 = 0$

$6x^2(x^3+8)=0$

$6x^2(x+2)(x^2-2x+4)=0$

$0, -2, 1 \pm i\sqrt{3}$

$x = \frac{2 \pm \sqrt{4-4\cdot4}}{2}$

$= \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2}$

$= 1 \pm i\sqrt{3}$

6. $8t^3 = 27$

$(2t)^3 - 3^3 = 0$

$(2t-3)(4t^2+6t+9)$

$\frac{3}{2}, \frac{2 \pm i\sqrt{17}}{4}$

$x = \frac{6 \pm \sqrt{36-4\cdot4\cdot9}}{2\cdot4}$

$= \frac{6 \pm \sqrt{-108}}{8}$

$= \frac{6 \pm 2i\sqrt{17}}{8}$

Factor & solve each equation.

7. $x^4 + 4x^2 = 32$

$(x^2)^2 + 4(x^2) - 32 = 0$

$(x^2 - 4)(x^2 + 8) = 0$

$(x+2)(x-2)(x^2+8) = 0$

$\boxed{2, -2, \pm 2i\sqrt{2}}$

8. $4x^5 - 8x^3 + 4x = 0$

$4x(x^4 - 2x^2 + 1) = 0$

$4x(x^2 - 1)(x^2 - 1) = 0$

$4x(x+1)(x-1)(x+1)(x-1) = 0$

$\boxed{0, -1 \text{ (mult 2)}, 1 \text{ (mult 2)}}$

9. $15x^2 - 17x + 4 = 0$

$(15x^2 - 20x)(+3x + 4) = 0$

$60 \quad -17$

$-20, +3$

$5x(3x-4) - 1(3x-4) = 0$

$(3x-4)(5x-1) = 0$ Factored form

$\boxed{x = \frac{4}{3}, \frac{1}{5}}$

10. $2x^3 - 5x^2 + 4x + 10 = 0$

$x^2(2x-5) + -2(2x-5) = 0$

$(2x-5)(x^2-2) = 0$

$x = \frac{5}{2}$

$x^2 = 2$

$x = \pm\sqrt{\ }$

II. Remainder and Factor Theorem

Use synthetic substitution to evaluate each function.

11. $P\left(-\frac{1}{3}\right); P(x) = 3x^3 - 5x^2 + 4x + 2$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & -5 & 4 & 2 \\ & \downarrow & -1 & 2 & -2 \\ \hline & 3 & -6 & 6 & 0 \end{array}$$

$\boxed{P\left(-\frac{1}{3}\right) = 0}$

12. $F\left(\frac{4}{5}\right); F(x) = 25x^2 + 16$

$$\begin{array}{r|rrr} \frac{4}{5} & 25 & 0 & 16 \\ & & 20 & 16 \\ \hline & 25 & 20 & 32 \end{array}$$

$\boxed{P\left(\frac{4}{5}\right) = 32}$

Use the remainder theorem to determine whether the given binomial is a factor of P(x). If yes, find the remaining factors and write P(x) in its fully factored form.

13. $(x-2); P(x) = x^3 + 5x^2 - 4x + 20$

$$\begin{array}{r|rrrr} 2 & 1 & 5 & -4 & 20 \\ & \downarrow & 2 & 14 & 20 \\ \hline & 1 & 7 & 10 & 40 \end{array}$$

No, Not a Factor, remainder $\neq 0$.

14. $(x-4); P(x) = x^3 - 10x^2 + 32x - 32$

$$\begin{array}{r|rrrr} 4 & 1 & -10 & 32 & -32 \\ & & 4 & -24 & 32 \\ \hline & 1 & -6 & 8 & 0 \end{array}$$

yes! remainder = 0

$P(x) = (x-4)(x^2 - 6x + 8)$

$P(x) = (x-4)(x-2)(x-4)$ ← Factored form

15. $(x+6); P(x) = x^3 - 27x + 54$

$$\begin{array}{r|rrrr} -6 & 1 & 0 & -27 & 54 \\ & & -6 & 36 & -54 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

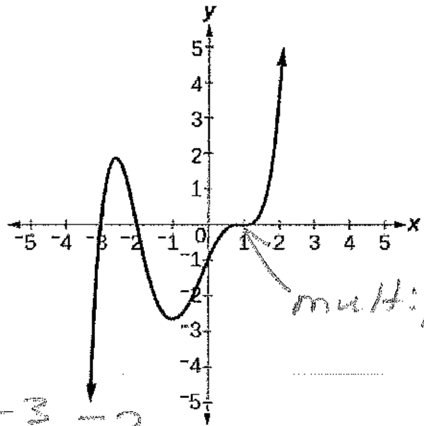
yes! remainder = 0.

$P(x) = (x+6)(x^2 - 6x + 9)$

$= (x+6)(x-3)(x-3)$ ← Factored form

Write the equation for each graph using zeros and multiplicities in factored form.

16



multiplicity = 3

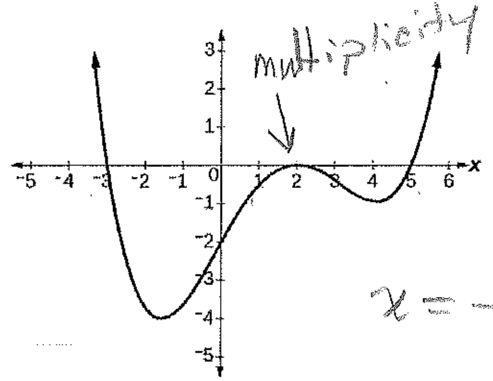
$y = -3, -2$

1, mult 3

$f(x) = (x-1)^3(x+3)(x+2)$

17

multiplicity = 2



$x = -3, 2, 5$

$f(x) = (x+3)(x-2)^2(x-5)$

Sketch the graph of the given polynomials using multiplicities.

18

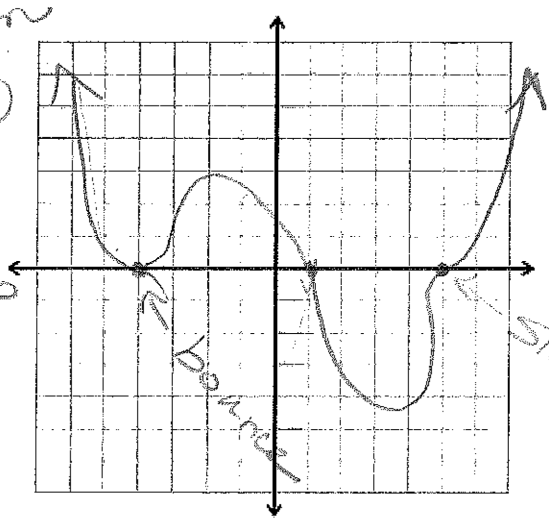
20. $f(x) = (x-5)^3(x-1)(x+4)^2$

even

LC (+)

E.B.

up/up



bounce

slide

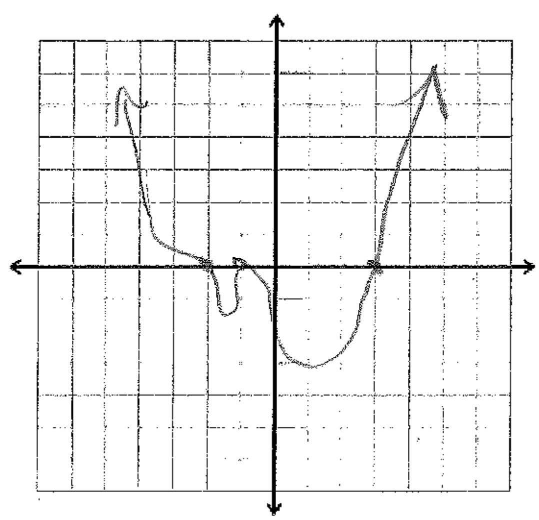
5 (mult 3), 1, -4 (mult 2)

"slide" "bounce"

19

21. $f(x) = (x+1)^2(x-3)(x+2)^3$

ev



$x = -1$ (mult 2) bounce

$= 3$

$= -2$ (mult 3) slide

III. Fundamental Theorem of Algebra

Use calc.

State the number of roots, based on the degree of the equation. Identify any real roots and use synthetic division to reduce the polynomial to a quadratic. Factor & solve for the remaining roots. State any multiplicity of each root.

20 16. $x^4 - 4x^3 + 29x^2 - 100x + 100 = 0$ 4 roots

$x = 2$ (from calculator) (mult 2) because of "bounce"

```
2 | 1  -4  29  -100   100
        2  -4    50 -100
2 | 1  -2  25   -50 |  0
        2   0    50
    1   0  25 |  0
```

$(x-2)^2(x^2+25) = 0$

$x^2 = -25$

$x = \pm 5i$

Solns:
$x = 2$ (mult 2)
$x = 5i$
$x = -5i$

21 17. $x^5 - 3x^4 + 15x^3 - 37x^2 + 36x - 12 = 0$

$x = 1$ (from calculator) (mult 3 b/c of "slide")

```
1 | 1  -3  15  -37  36  -12
        1  -2   13 -24   12
1 | 1  -2  13  -24  12 |  0
        1  -1   12 -12
1 | 1  -1  12  -12 |  0
        1   0   12
    1   0  12 |  0
```

$(x-1)^3(x^2+12) = 0$

$x^2 + 12 = 0$

$x^2 = -12$

$x = \pm 2i\sqrt{3}$

Solns: $x = 1$ (mult 3)
$= 2i\sqrt{3}$
$= -2i\sqrt{3}$

IV. Writing equations from roots

State the degree, then write the *simplest* polynomial function with the given zeros.

22
~~18.~~ 4 **and** $\sqrt{3}$

$f(x) = (x-4)(x+\sqrt{3})(x-\sqrt{3})$

degree: 3

$= (x-4)(x^2-3)$

$= x^3 - 3x - 4x^2 + 12$

$\boxed{f(x) = x^3 - 4x^2 - 3x + 12}$

23
~~19.~~ $\sqrt{2}$ **and** $5i$, $-5i$, $-\sqrt{2}$

$(x-5i)(x+5i)(x-\sqrt{2})(x+\sqrt{2})$

$(x^2+25)(x^2-2)$

$x^4 - 2x^2 + 25x^2 - 50$

$\boxed{x^4 + 23x^2 - 50 = f(x)}$

Degree = 4

24
~~20.~~ -1 **and** $2i$, $-2i$

Degree: 3

$(x+1)(x-2i)(x+2i)$

$(x+1)(x^2+4)$

$x^3 + 4x + x^2 + 4$

$\boxed{x^3 + x^2 + 4x + 4 = f(x)}$

25
~~21.~~ 2 **and** $1-2i$, $1+2i$

Degree: 3

$x = 1 \pm 2i$

$(x-2)$

$(x-1)^2 = (\pm 2i)^2$

$x^2 - 2x + 1 =$